Unit 1 on a Page

Principles and Definitions
If you have had me or Dr. Bourgeois for P131, you are familiar with a distinction I make between *principles* and *definitions*. Principles are the fundamental rules of the Universe that describe how things work. Definitions, on the other hand, simply describe a quantity. For example,

$$\vec{p} = m\vec{v}$$

is the *definition* of momentum for a massive particle; this equation offers no deep foundational insights on how the universe works. We physicists simply noted that the quantity

$$m\vec{v}$$

came up a lot and we gave it a name

$\vec{p}$mover

. In order to describe how the Universe works, principles will often involve multiple definitions. Note, sometimes a principle or definition has an equation, other times it is just stated in words! This connects to Physics Goals 1 and 2 for this course.

To help get those of you who may not be used to this distinction acquainted and to help organize the huge amount of factual information in this particular unit, I will list the principles for this unit. You can quickly see how much shorter this list is than the list above. If fact, not every section in the Topics and Objectives has a principle!

Basic Properties of Light

- Light in a vacuum always travels at the speed of light

Basic Properties of Waves

- Fundamental connection between wavelength, frequency, and wave speed:

$$v = \lambda \bullet f$$

- Amplitude is independent of frequency

Basics of Energy

- Energy is conserved:

$$\Delta E = Q + W$$

(in this class

movermover

and

movermover

will most always be zero)

Wave Particle Duality

- You can convert from the wave picture to the particle picture through the de Broglie relation:

$$p = \frac{h}{\lambda}$$

. NOTE, this is NOT

$$E = \frac{hc}{\lambda}$$

that you learned in chemistry. The equation

$$E = \frac{hc}{\lambda}$$

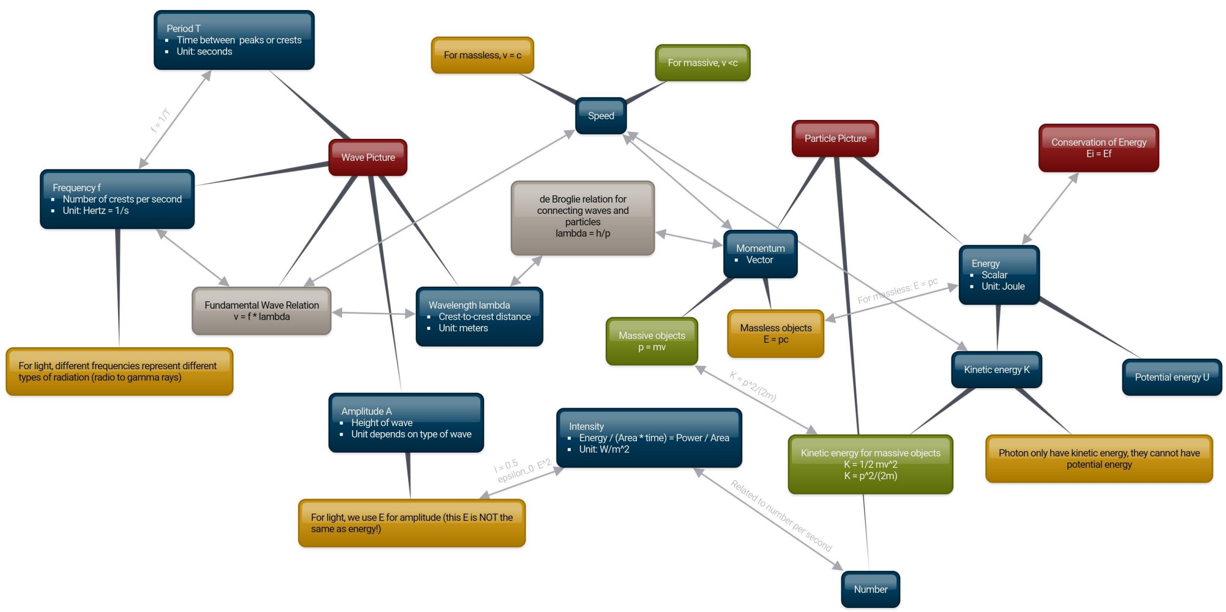
only applies for light while

$$p = \frac{h}{\lambda}$$

applies to BOTH light and electrons.
- The probability of finding a particle in a given location is proportional to the square of the amplitude

Standing Waves

- The wave must "fit" in the box or on the ring. For a box, this means that the box must be an integer number of ½ wavelengths. For a ring, an integer number of waves must fit around the ring.



Exercise:

UMASS AMHERST Instructor's Notes

Problem:

This unit has a lot of ideas that can be connected in many different ways. A good way to represent such information is in a concept map such as the one shown above, which is also available at the link [here.]

In the map above:

- UMass maroon bubbles are big ideas
- Yellow bubbles apply to massless particles like light
- Green bubbles apply to massive particles like electrons

I would recommend printing a copy for use in class!

Introduction to the Unit

In this unit, we will follow our ontological framework and begin exploring what light and electrons are by listing some of their basic properties. One of the most famous properties of both light and electrons is known as wave-particle duality: sometimes they behave like particles and sometimes they behave like waves. However, electrons and light are neither particles nor waves: they are a completely new type of object with properties of both. This duality is a reflection of the fact that both light and electrons do not obey the laws of Classical Physics that you learned in Physics 131, they are too small. Instead, electrons and light obey Quantum Mechanics. The way I would recommend that you think about the relationship between classical physics and quantum mechanics is in the paradigm of physics striving for ever-more-accurate approximations to reality. Classical mechanics is a good enough approximation to get people to the moon (they did it!). When things get small, you need a better approximation: quantum mechanics. There is a principle, the correspondence principle, which states the quantum mechanics must reproduce classical mechanics for large objects.

In this unit we will explore both the wave and particle natures of light and electrons. First however, we must define to ourselves what waves and particles are! Following our ontological framework, we will therefore need to look at some of the basic properties that characterize waves and particles in general. Thus, we will begin with some review of particles from physics 131 and then a discussion of waves, which may be familiar to some of you. Once we have defined waves and particles through listing their properties we will explore how these properties manifest for electrons and light.

As you read, you MUST keep in mind that light and electrons are neither particles nor waves. They are something completely new (quantum mechanical objects) that you have zero previous experience with. Particles and waves are simply ways of visualizing these objects in ways our brains can understand. Neither picture is 100% correct. The correct approach is to jump back-and-forth between these two pictures

What We Will Do in Class

This idea of electrons and light being neither particles no waves but having properties of both is a hard one to get used to. In class, we will spend a lot of time practicing jumping between the wave and particle pictures: seeing the benefits that each picture can bring in various situations. The goal is that, through practice, you become more comfortable with bouncing back and forth between pictures as the situation demands.

After a bit of practice with moving between the wave picture and particle pictures of light and electrons, we will combine this understanding with one of the most important ideas in physics: conservation of energy. Thinking about this fundamental principle in conjunction with the fundamentally quantum nature of light and electrons will allow us to understand many different phenomena such as, “Why do electrons in atoms have defined energy levels?” You know from chemistry courses that they do. Our goal is to explain w

What You Should Get Out of the Prep

In order to explore these ideas in class, you need to have a grasp on the basic terminology of waves and particles. You need to know that particles are characterized by their energy, momentum, and number while waves are described by their wavelength and frequency/period, and amplitude. Both waves and particles can be characterized by a speed: a critical fact for converting between the wave and particle pictures. The following chapters will refresh energy and momentum from 131 and introduce the needed concepts for waves: amplitude, frequency, and wavelength. You need to know what all these terms mean, the basic formulas for them, and how they are connected

Also in this unit, some of the basic properties of light and electrons. We will establish electrons via a tour of the atom (probably review for most of you). We will also introduce anti-matter: a type of matter identical to the normal matter with which you are familiar in every respect with two exceptions. First, anti-matter has the opposite charge from normal matter (anti-electrons have positive charge). Second, when matter and anti-matter collide, the result is light. While there are other interesting questions about anti-matter (why isn't it everywhere?) those two points are all you need to know from the prep. With regards to light, we will introduce the different kinds of light (radio, infrared, …) and the particle of light: the photon. Some of the basic properties of the photon will be introduced. The most important of which you need for the prep are the fact that the mass of the photon is zero and the fact that it always travels at the speed of light c. Finally, we will also explore de Broglie's relationship on how to connect wave and particle picture

The last topic in this unit's preparation you need to be familiar with is the idea of intensity: energy per area per time. The text will introduce the idea of power (energy per time) and its unit the Watt. This concept will also be important for converting between the wave and particle pictures.

This is a lot of information. Remember, we are not expecting mastery of it all and we are certainly not expecting you to have a complete picture of how everything Just make sure you know the definitions, formulas, and units for everything. To help keep you focused on what is important, there

is a summary table below (only focusing on the facts for your quiz) as well as some flashcards at Quizlet..

General Waves	**Properties:** • **Amplitude** A • **Wavelength** λ • **Period** T • **Frequency** $f = \frac{1}{T}$ **Speed** $v = \lambda \bullet f$	**Intensity:** • **Energy per area per time** $I = \frac{E}{A \bullet t} = \frac{P}{A}$

<table>
<tr>
<td>General Particles</td>
<td>Properties:
Energy
$$E$$
Momentum
$$\vec{p}$$
Speed
$$v$$
Number
$$N$$</td>
<td>Convert to Waves:
Wave-particle conversion is done through the de Broglie relation
$$p = \frac{h}{\lambda}$$
Can also convert via intensity where the intensity is also related to number per area per time.</td>
</tr>
</table>

Light	**Waves:** • Travel at c • Different frequencies → different kinds of radiation • Amplitude E (NOT energy!) is related to intensity $I = \frac{1}{2}\varepsilon_0 E^2$	**Particles (Photons):** • Massless • Travel at c • $p = \frac{E}{c}$

<table>
<tr>
<td>Electrons</td>
<td>Waves:
You only need the general de Broglie relation
$$p = \frac{h}{\lambda}$$</td>
<td>Particles:
Momentum
$$\overrightarrow{p} = m \bullet \overrightarrow{v}$$
Kinetic energy
$$K = \frac{1}{2}mv^2 = 2m \bullet \overrightarrow{p}$$
Mass
$$m_e = 9.11 \times 10^{-31} kg$$
Charge
$$q_e = 1.67 \times 10^{-19} C$$</td>
</tr>
</table>

Static Electricity and Charge

- Define electric charge, and describe how the two types of charge interact.
- Describe three common situations that generate static electricity.
- State the law of conservation of charge.

Exercise:

UMASS AMHERST Instructor's Notes

Problem:
Your quiz will expect you to know the following basic characteristics of static electricity:

- The effects of static electricity are explained by a physical quantity not previously introduced, called electric charge.
- There are only two types of charge, one called positive and the other called negative.
- Like charges repel, whereas unlike charges attract.
- The force between charges decreases with distance.

Borneo amber was mined in Sabah, Malaysia, from shale-sandstone-mudstone veins.

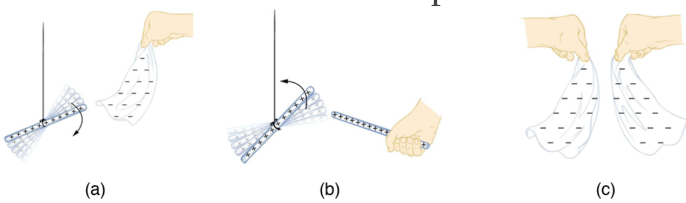
When a piece of amber is rubbed with a piece of silk, the amber gains more electrons, giving it a net negative charge. At the same time, the silk, having lost electrons, becomes positively charged. (credit: Sebakoamber, Wikimedia Commons)

What makes plastic wrap cling? Static electricity. Not only are applications of static electricity common these days, its existence has been known since ancient times. The first record of its effects dates to ancient Greeks who noted more than 500 years B.C. that polishing amber temporarily enabled it to attract bits of straw (see [link]). The very word *electric* derives from the Greek word for amber (*electron*).

Many of the characteristics of static electricity can be explored by rubbing things together. Rubbing creates the spark you get from walking across a wool carpet, for example. Static cling generated in a clothes dryer and the attraction of straw to recently polished amber also result from rubbing. Similarly, lightning results from air movements under certain weather conditions. You can also rub a balloon on your hair, and the static electricity created can then make the balloon cling to a wall. We also have to be cautious of static electricity, especially in dry climates. When we pump gasoline, we are warned to discharge ourselves (after sliding across the seat) on a metal surface before grabbing the gas nozzle. Attendants in hospital operating rooms must wear booties with aluminum foil on the bottoms to avoid creating sparks which may ignite the oxygen being used.

How do we know there are two types of **electric charge**? When various materials are rubbed together in controlled ways, certain combinations of materials always produce one type of charge on one material and the opposite type on the other. By convention, we call one type of charge “positive”, and the other type “negative.” For example, when glass is

rubbed with silk, the glass becomes positively charged and the silk negatively charged. Since the glass and silk have opposite charges, they attract one another like clothes that have rubbed together in a dryer. Two glass rods rubbed with silk in this manner will repel one another, since each rod has positive charge on it. Similarly, two silk cloths so rubbed will repel, since both cloths have negative charge. [link] shows how these simple materials can be used to explore the nature of the force between charges.



A glass rod becomes positively charged when rubbed with silk, while the silk becomes negatively charged. (a) The glass rod is attracted to the silk because their charges are opposite. (b) Two similarly charged glass rods repel. (c) Two similarly charged silk cloths repel.

More sophisticated questions arise. Where do these charges come from? Can you create or destroy charge? Is there a smallest unit of charge? Exactly how does the force depend on the amount of charge and the distance between charges? Such questions obviously occurred to Benjamin Franklin and other early researchers, and they interest us even today.

Exercise:

UMASS AMHERST Instructor's Notes

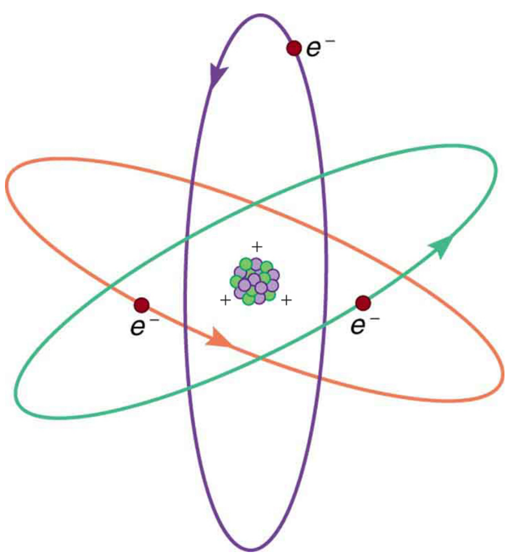
Problem:

The structure of the atom will be discussed in more detail in a later section.

Charge Carried by Electrons and Protons

Franklin wrote in his letters and books that he could see the effects of electric charge but did not understand what caused the phenomenon. Today we have the advantage of knowing that normal matter is made of atoms, and that atoms contain positive and negative charges, usually in equal amounts.

[link] shows a simple model of an atom with negative **electrons** orbiting its positive nucleus. The nucleus is positive due to the presence of positively charged **protons**. Nearly all charge in nature is due to electrons and protons, which are two of the three building blocks of most matter. (The third is the neutron, which is neutral, carrying no charge.) Other charge-carrying particles are observed in cosmic rays and nuclear decay, and are created in particle accelerators. All but the electron and proton survive only a short time and are quite rare by comparison.



This simplified (and not to scale) view of an atom is called the planetary model of the atom. Negative electrons orbit a much heavier positive nucleus, as the planets orbit the much heavier sun. There the similarity ends, because forces in the atom are electromagnetic, whereas those in the planetary system are gravitational. Normal macroscopic amounts of matter contain immense numbers of atoms and molecules and, hence, even greater numbers of individual negative and positive charges.

The charges of electrons and protons are identical in magnitude but opposite in sign. Furthermore, all charged objects in nature are integral multiples of this basic quantity of charge, meaning that all charges are made of combinations of a basic unit of charge. Usually, charges are formed by combinations of electrons and protons. The magnitude of this basic charge is

Equation:

$$\mid q_e \mid = 1.60 \times 10^{-19}\ \text{C}.$$

The symbol q is commonly used for charge and the subscript e indicates the charge of a single electron (or proton).

The SI unit of charge is the coulomb (C). The number of protons needed to make a charge of 1.00 C is

Equation:

$$1.00\text{ C}\times\frac{1\text{ proton}}{1.60\times10^{-19}\text{ C}}=6.25\times10^{18}\text{ protons.}$$

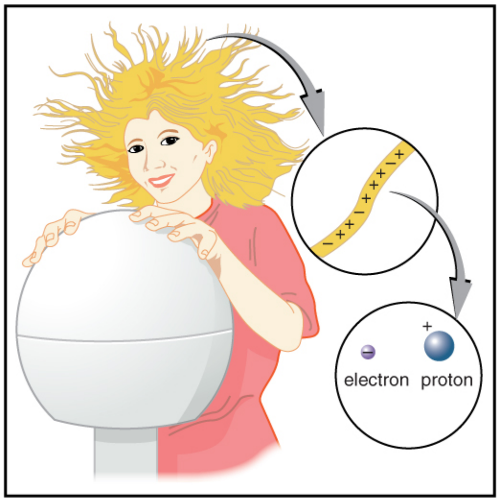
Similarly, 6.25×10^{18} electrons have a combined charge of −1.00 coulomb. Just as there is a smallest bit of an element (an atom), there is a smallest bit of charge. There is no directly observed charge smaller than $\mid q_e \mid$, and all observed charges are integral multiples of $\mid q_e \mid$.

Note:

Things Great and Small: The Submicroscopic Origin of Charge

With the exception of exotic, short-lived particles, all charge in nature is carried by electrons and protons. Electrons carry the charge we have named negative. Protons carry an equal-magnitude charge that we call positive. (See [link].) Electron and proton charges are considered fundamental building blocks, since all other charges are integral multiples of those carried by electrons and protons. Electrons and protons are also two of the three fundamental building blocks of ordinary matter. The neutron is the third and has zero total charge.

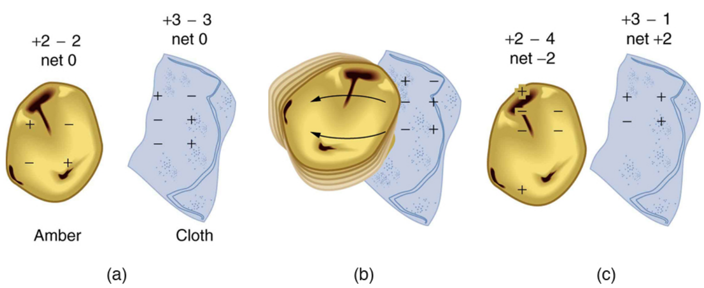
[link] shows a person touching a Van de Graaff generator and receiving excess positive charge. The expanded view of a hair shows the existence of both types of charges but an excess of positive. The repulsion of these positive like charges causes the strands of hair to repel other strands of hair and to stand up. The further blowup shows an artist’s conception of an electron and a proton perhaps found in an atom in a strand of hair.



When this person touches a Van de Graaff generator, she receives an excess of positive charge, causing her hair to stand on end. The charges in one hair are shown. An artist's conception of an electron and a proton illustrate the particles carrying the negative and positive charges. We cannot really see these particles with visible light because they are so small (the electron seems to be an infinitesimal point), but we know a great deal about their measurable properties, such as the charges they carry.

Separation of Charge in Atoms

Charges in atoms and molecules can be separated—for example, by rubbing materials together. Some atoms and molecules have a greater affinity for electrons than others and will become negatively charged by close contact in rubbing, leaving the other material positively charged. (See [link].) Positive charge can similarly be induced by rubbing. Methods other than rubbing can also separate charges. Batteries, for example, use combinations of substances that interact in such a way as to separate charges. Chemical interactions may transfer negative charge from one substance to the other, making one battery terminal negative and leaving the first one positive.



When materials are rubbed together, charges can be separated, particularly if one material has a greater affinity for electrons than another. (a) Both the amber and cloth are originally neutral, with equal positive and negative charges. Only a tiny fraction of the charges are involved, and only a few of them are shown here. (b) When rubbed together, some negative charge is transferred to the amber, leaving the cloth with a net positive charge. (c) When separated, the amber and cloth now have net charges, but the absolute value of the net positive and negative charges will be equal.

No charge is actually created or destroyed when charges are separated as we have been discussing. Rather, existing charges are moved about. In fact, in all situations the total amount of charge is always constant. This universally obeyed law of nature is called the **law of conservation of charge**.

Exercise:

UMASS AMHERST Instructor's Notes

Problem:

The Law of Conservation of Charge is so critical that we will explore it more in a later section.

> **Note:**
> Law of Conservation of Charge
> Total charge is constant in any process.

In more exotic situations, such as in particle accelerators, mass, Δm, can be created from energy in the amount $\Delta m = \frac{E}{c^2}$. Sometimes, the created mass is charged, such as when an electron is created. Whenever a charged particle is created, another having an opposite charge is always created along with it, so that the total charge created is zero. Usually, the two particles are "matter-antimatter" counterparts. For example, an antielectron would usually be created at the same time as an electron. The antielectron has a positive charge (it is called a positron), and so the total charge created is zero. (See [link].) All particles have antimatter counterparts with opposite signs. When matter and antimatter counterparts are brought together, they completely annihilate one another. By annihilate, we mean that the mass of the two particles is converted to energy E, again obeying the relationship

$\Delta m = \frac{E}{c^2}$. Since the two particles have equal and opposite charge, the total charge is zero before and after the annihilation; thus, total charge is conserved.

Exercise:

UMASS AMHERST Instructor's Notes

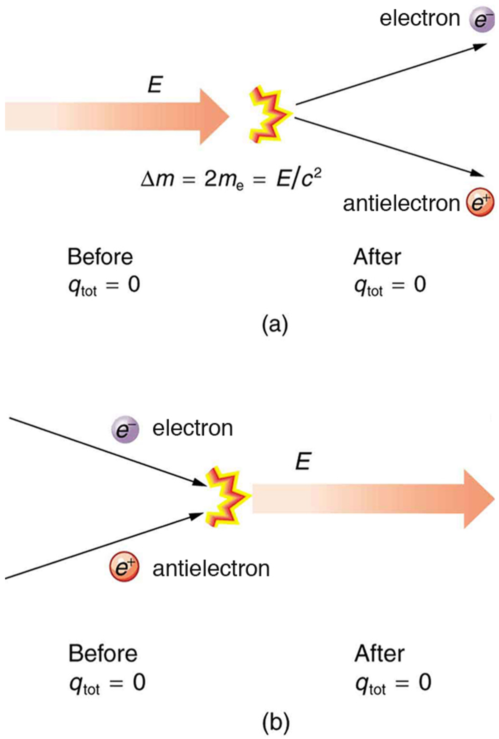
Problem:

We will be occasionally dealing with anti-matter in this class. You need to know that matter and antimatter are identical in mass, but opposite in charge: an anti-electron has a positive charge. You also need to know that when matter and anti-matter come together the result is pure energy.

Note:

Making Connections: Conservation Laws

Only a limited number of physical quantities are universally conserved. Charge is one—energy, momentum, and angular momentum are others. Because they are conserved, these physical quantities are used to explain more phenomena and form more connections than other, less basic quantities. We find that conserved quantities give us great insight into the rules followed by nature and hints to the organization of nature. Discoveries of conservation laws have led to further discoveries, such as the weak nuclear force and the quark substructure of protons and other particles.



(a) When enough energy is present, it can be converted into matter. Here the matter created is an electron–antielectron pair. (m_e is the electron's mass.) The total charge before and after this event is zero. (b) When matter and antimatter collide, they annihilate each other; the total charge is conserved at zero before and after the annihilation.

The law of conservation of charge is absolute—it has never been observed to be violated. Charge, then, is a special physical quantity, joining a very

short list of other quantities in nature that are always conserved. Other conserved quantities include energy, momentum, and angular momentum.

Section Summary

- There are only two types of charge, which we call positive and negative.
- Like charges repel, unlike charges attract, and the force between charges decreases with the square of the distance.
- The vast majority of positive charge in nature is carried by protons, while the vast majority of negative charge is carried by electrons.
- The electric charge of one electron is equal in magnitude and opposite in sign to the charge of one proton.
- An ion is an atom or molecule that has nonzero total charge due to having unequal numbers of electrons and protons.
- The SI unit for charge is the coulomb (C), with protons and electrons having charges of opposite sign but equal magnitude; the magnitude of this basic charge $\mid q_e \mid$ is
  Equation:

$$\mid q_e \mid = 1.60 \times 10^{-19}\text{ C}.$$

- Whenever charge is created or destroyed, equal amounts of positive and negative are involved.
- Most often, existing charges are separated from neutral objects to obtain some net charge.
- Both positive and negative charges exist in neutral objects and can be separated by rubbing one object with another. For macroscopic objects, negatively charged means an excess of electrons and positively charged means a depletion of electrons.
- The law of conservation of charge ensures that whenever a charge is created, an equal charge of the opposite sign is created at the same time.

Glossary

electric charge

a physical property of an object that causes it to be attracted toward or repelled from another charged object; each charged object generates and is influenced by a force called an electromagnetic force

law of conservation of charge

states that whenever a charge is created, an equal amount of charge with the opposite sign is created simultaneously

electron

a particle orbiting the nucleus of an atom and carrying the smallest unit of negative charge

proton

a particle in the nucleus of an atom and carrying a positive charge equal in magnitude and opposite in sign to the amount of negative charge carried by an electron

A Deeper Structure of the Atom
Exercise:

UMASS AMHERST Instructor's Notes

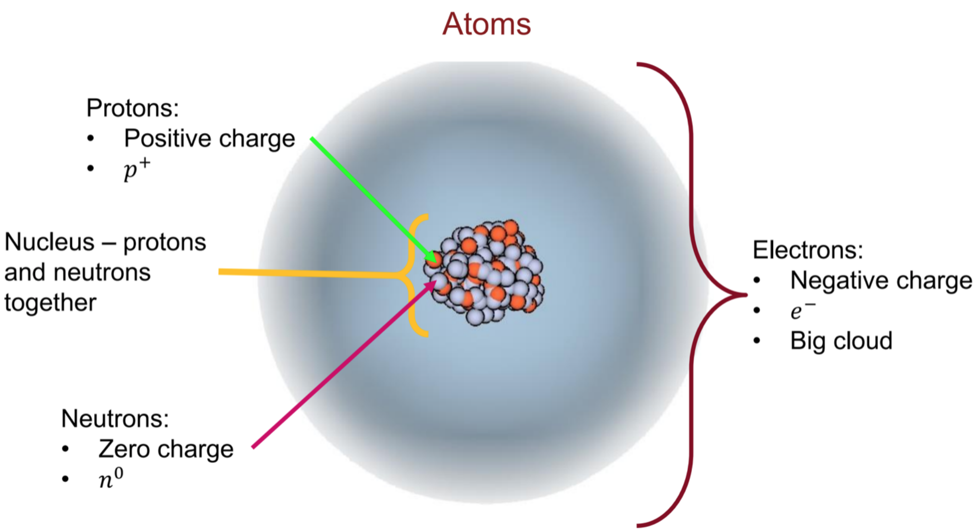
Problem:

This section is also available as a video on the UMass Physics 13X YouTube page. The link can be found here.
The big important features of the atom are:

- electrons and protons are charged, neutrons are not; the size of the charge on the electron and proton is the same, but the signs are different, so same magnitude different sign
- opposites attract and that is what holds the atom together
- protons and neutrons have the same mass, and electrons are way lighter
- protons and neutrons made of stuff, electrons are fundamental
- the nucleus is super tiny relative to atom

Those are your key takeaways from this section.

You should be familiar with the basic structure of the atom, but as a review, in the middle of the atom the positively-charged protons and neutrons are huddled together in the nucleus.



The basic structure of the Atom.

However, you might not be familiar with the related symbols. This symbol,

$$p^+$$

, means proton, P for proton, and then plus to remind us that it has a positive electrical charge (If you're wondering, can you have a negatively charged proton? Yes, it's called an antimatter proton.) You also have neutrons. Neutron has zero charge, so it's symbol is

$$n^0$$

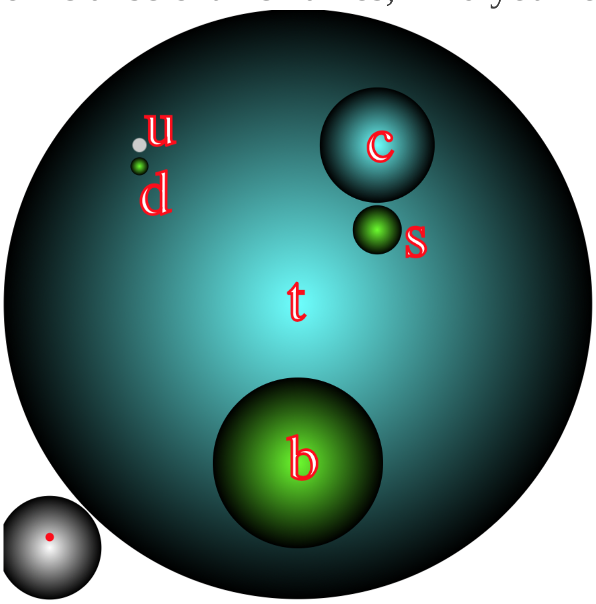
. Those are huddled together in the nucleus, and then surrounding the nucleus is a big cloud of negatively charged electrons, so we will use the symbol

$$e^-$$

. for electron. It's the attraction between the positively charged protons and the negatively charged electrons that sort of hold the entire atom together, and how that all works will be the emphasis of Unit 3.

Electrons are a big focus of this course, so it is worth discussing what they are made of. To our best of our knowledge they are not made up of anything. They are fundamental, we have been trying to smash them apart, but no luck. Maybe it's possible, but no one's been able to do it. If it is possible, we haven't hit it hard enough. That's very much the particle physics approach to everything- hit it harder and see if it breaks. So, electrons are fundamental building blocks- as far as we know they're not made up of anything.

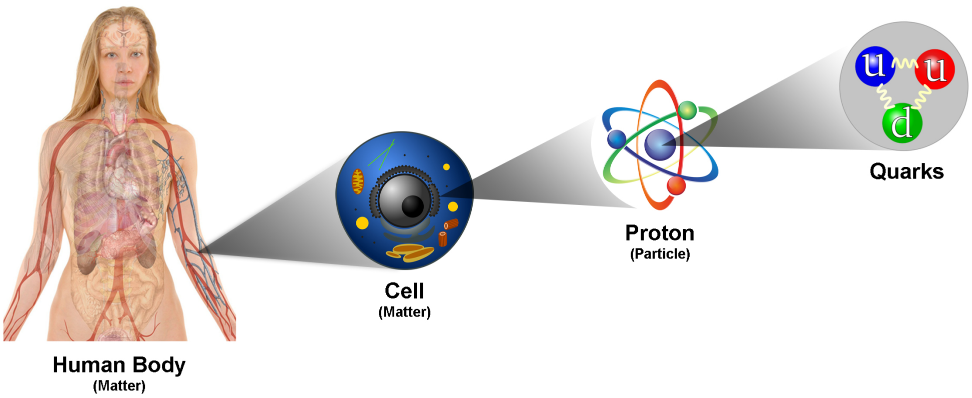
Protons and neutrons on the other hand, are a lot more fun, because they are made up of smaller pieces called quarks. There are six kinds of quarks, we have ‘up’, ‘down’, ‘strange’, ‘charm’, ‘top’, and ‘bottom’. Those are their official scientific names, I kid you not.



'u' corresponds to 'up' quarks, 'd' corresponds to 'down' quarks, 's' corresponds to 'strange' quarks, 'c' corresponds to 'charm' quarks, 't' corresponds to 'top' quarks, and 'b' corresponds to 'bottom' quarks. (credit: Incnis Mrsi, Wikipedia)

In the figure above, the sizing of the circles shows you the masses, how heavy this stuff is. The ‘top’ quark, the heaviest of the known quarks, actually has about the same mass as an entire atom. It's quite a heavy little thing. Three of these quarks, 'top', 'bottom', and 'charm', are actually heavier than protons.

But, those are all heavier than protons and neutrons, so what makes up a proton in a neutron? Protons and neutrons are made up of just these two ‘ups’ and ‘downs’. So, a proton is made up of two ‘up’ quarks and one ‘down’ quark, a neutron on the other hand is two ‘down’ quarks and an ‘up’ quark.



Quarks make up both protons and neutrons. (credit: Wikipedia)

Now you start doing math, so you need the proton to have +1 charge, the neutron to have 0 charge, you have two ‘ups’ and a ‘down’, and two ‘downs’ and an ‘up’. If you play with those numbers, what do you get? You have that ‘up’ quarks have a charge of

$$\frac{2}{3}$$

that of the proton, and down quarks are

$$- - -\frac{1}{3}$$

that of the proton. And this works out, think ‘up’ ‘up’ ‘down’ so that's

$$\frac{2}{3}+\frac{2}{3}-\frac{1}{3}=+1$$

,the charge of a proton, and it works out.

Similarly, think,

$$-\frac{1}{3}-\frac{1}{3}+\frac{2}{3}=0$$

, the charge of a neutron, they work out.
Exercise:

UMASS AMHERST Instructor's Notes

Problem:

What's your big takeaway for this? Electrons are fundamental to our knowledge and cannot be broken apart. Protons and neutrons are made up of smaller stuff.

The other key things to know about atoms: protons and neutrons are very very close to the same mass, but neutrons are a tiny bit heavier, but not by much. Electrons on the other hand are way lighter than protons or neutrons. In fact, the electron is the lightest known particle to have electric charge with a mass of

$$9.11\times10^{-31}$$

kilograms. Protons on the other hand, are much bigger,

$$1.67\times10^{-27}$$

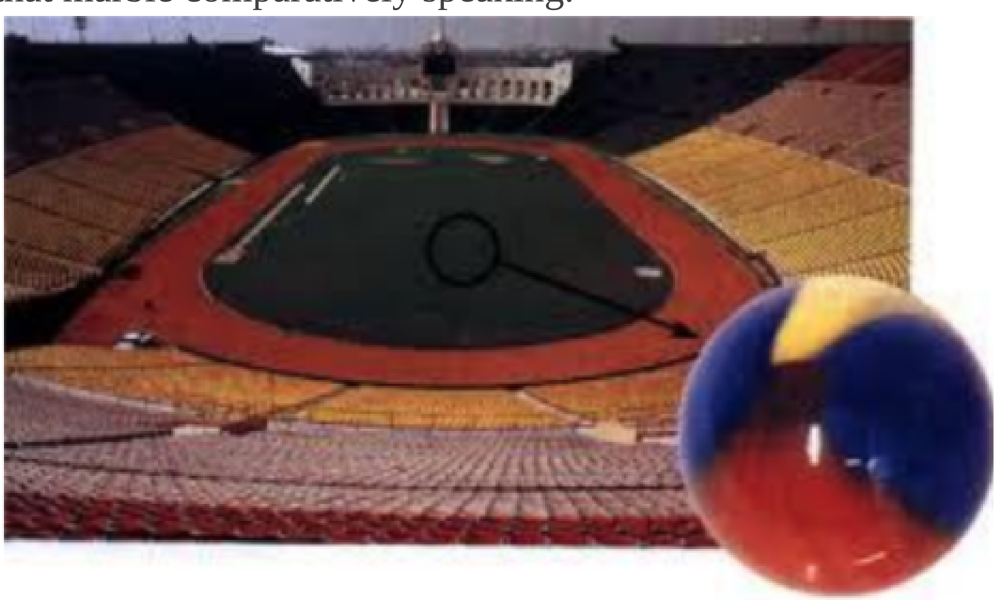
kilograms.
Exercise:

UMASS AMHERST Instructor's Notes

Problem:

What should you take away from this? Protons are way bigger than electrons, roughly 2,000 times (1836 times to be specific).

If you prefer to think about atoms instead of protons and neutrons, you can think about a Helium atom, you know there are two protons, two neutrons, and two electrons. The electrons make up 0.03% of the mass of helium. Electrons don't weigh squat. They don't really matter as far as mass goes. While the nucleus has most of the mass, it doesn't take up a lot of space. The standard analogy that people make is if you blow up the atom to the size of a large college football stadium, bigger than ours, the nucleus is roughly the size of a marble. Atoms are a whole bunch of empty nothing. The nucleus is about the size of a marble, but it is 99.97% of the mass is in that marble comparatively speaking.



If the atom is the size of a football stadium, the nucleus

is the size of a marble. (credit: Wikipedia)

More on Conservation of Electric Charge
Exercise:

UMASS AMHERST Instructor's Notes

Problem:

This section is also available as a video on the UMass Physics 13X YouTube page. The link can be found here.

We are going to do examples. Let's say I rub a plastic rod with some fur, basic friction, and I move about

$$10^6$$

electrons from the fur to the rod. What's the charge of everything when we're done?

Well we have

$$10^6$$

electrons going from the fur to the rod. Now charge has to be conserved. The rod is clearly going to end up with a negative charge from the added electrons, but since charge has to be conserved, and I took those electrons from the fur, my fur has to have an equal positive charge. Charges had to come from somewhere, and they came from the fur.

Now let's talk about how much charge. We know it's

$$10^6$$

electrons worth. We know it's

$$1.602\times10^{-19}$$

coulombs for each electron. So, the rod will have a charge of

$$1.602 \times 10^{-13}$$

coulombs, and

$$q$$

is the letter we use for charge. We use

$$q$$

and the charge on the rod is going to be negative and the fur will have the exact same positive charge.

Conceptual Questions

Exercise:

Problem:

There are very large numbers of charged particles in most objects. Why, then, don't most objects exhibit static electricity?

Exercise:

Problem:

Why do most objects tend to contain nearly equal numbers of positive and negative charges?

Problems and Exercises

Exercise:

Problem:

Common static electricity involves charges ranging from nanocoulombs to microcoulombs. (a) How many electrons are needed to form a charge of –2.00 nC (b) How many electrons must be removed from a neutral object to leave a net charge of $0.500 \ \mu\text{C}$?

Solution:

(a) 1.25×10^{10}

(b) 3.13×10^{12}

Exercise:

Problem:

If 1.80×10^{20} electrons move through a pocket calculator during a full day’s operation, how many coulombs of charge moved through it?

Exercise:

Problem:

To start a car engine, the car battery moves 3.75×10^{21} electrons through the starter motor. How many coulombs of charge were moved?

Solution:

-600 C

Exercise:

Problem:

A certain lightning bolt moves 40.0 C of charge. How many fundamental units of charge $\mid q_e \mid$ is this?

What is a Particle

Exercise:

Problem:

We talked about particles a lot in Physics 131 in the point mass approximation, but it's probably best that we flush out our definitions.

What is a particle? The simplest image of a particle is probably just a ball. What properties apply to all particles?

In its most generic sense, a particle is a chunk of stuff. It exists in a particular place and at a particular time and a particle doesn't go around corners. If I throw a ball at a door, it'll either go through the door or bounce back, it won't curve around it. Particles can, but do not necessarily have to, have mass, we will talk about a massless particle in a later section. But all particles can be thought of as having momentum, that quantity from 131 of mass times velocity. Particles can also be thought of as having energy.

Linear Momentum and Force

- Calculate the momentum for any object
- Recall that momentum is a vector
- From the change in momentum, compute the average force

Exercise:

UMASS AMHERST Instructor's Notes

Problem:

This section is a review from Physics 131, but we will continue to use these ideas throughout the Unit.
Your Quiz will Cover

- Calculate the momentum for any object
- Recall that momentum is a vector
- From the change in momentum, compute the average force

Linear Momentum

The scientific definition of linear momentum is consistent with most people's intuitive understanding of momentum: a large, fast-moving object has greater momentum than a smaller, slower object. **Linear momentum** is defined as the product of a system's mass multiplied by its velocity. In symbols, linear momentum is expressed as
Equation:

$$\mathbf{p} = m\mathbf{v}.$$

Momentum is directly proportional to the object's mass and also its velocity. Thus the greater an object's mass or the greater its velocity, the

greater its momentum. Momentum $\mathbf{p}$ is a vector having the same direction as the velocity $\mathbf{v}$. The SI unit for momentum is $\mathrm{kg} \cdot \mathrm{m/s}$.

Note:
Linear Momentum
Linear momentum is defined as the product of a system's mass multiplied by its velocity:
Equation:

$$\mathbf{p} = m\mathbf{v}.$$

Exercise:

UMASS AMHERST Instructor's Notes

Problem:

The main focus for this section is the definition of momentum above, as well as the calculation of momentum. The following is a good example of what we expect you to be able to do with regards to the calculation. Also, pay attention to the discussion in the example as well, as it talks about how both mass and velocity can affect momentum.

Example:
Calculating Momentum: A Football Player and a Football
(a) Calculate the momentum of a 110-kg football player running at 8.00 m/s. (b) Compare the player's momentum with the momentum of a hard-thrown 0.410-kg football that has a speed of 25.0 m/s.

Strategy

No information is given regarding direction, and so we can calculate only the magnitude of the momentum, p. (As usual, a symbol that is in italics is a magnitude, whereas one that is italicized, boldfaced, and has an arrow is a vector.) In both parts of this example, the magnitude of momentum can be calculated directly from the definition of momentum given in the equation, which becomes

Equation:

$$p = \mathrm{mv}$$

when only magnitudes are considered.

Solution for (a)

To determine the momentum of the player, substitute the known values for the player's mass and speed into the equation.

Equation:

$$p_{\text{player}} = (110\ \text{kg})(8.00\ \text{m/s}) = 880\ \text{kg} \cdot \text{m/s}$$

Solution for (b)

To determine the momentum of the ball, substitute the known values for the ball's mass and speed into the equation.

Equation:

$$p_{\text{ball}} = (0.410\ \text{kg})(25.0\ \text{m/s}) = 10.3\ \text{kg} \cdot \text{m/s}$$

The ratio of the player's momentum to that of the ball is

Equation:

$$\frac{p_{\text{player}}}{p_{\text{ball}}} = \frac{880}{10.3} = 85.9.$$

Discussion

Although the ball has greater velocity, the player has a much greater mass. Thus the momentum of the player is much greater than the momentum of the football, as you might guess. As a result, the player's motion is only slightly affected if he catches the ball. We shall quantify what happens in such collisions in terms of momentum in later sections.

Exercise:

UMASS AMHERST Instructor's Notes

Problem:

All you need to know from this section is the definition of momentum. The following connection to Newton's 2nd Law is just to help you put this info into context.

Momentum and Newton’s Second Law

The importance of momentum, unlike the importance of energy, was recognized early in the development of classical physics. Momentum was deemed so important that it was called the “quantity of motion.” Newton actually stated his **second law of motion** in terms of momentum: The net external force equals the change in momentum of a system divided by the time over which it changes. Using symbols, this law is

Equation:

$$\mathbf{F}_{\text{net}} = \frac{\Delta\mathbf{p}}{\Delta t},$$

where $\mathbf{F}_{\text{net}}$ is the net external force, $\Delta\mathbf{p}$ is the change in momentum, and Δt is the change in time.

Note:
Newton’s Second Law of Motion in Terms of Momentum
The net external force equals the change in momentum of a system divided by the time over which it changes.
Equation:

$$\mathbf{F}_{\text{net}} = \frac{\Delta \mathbf{p}}{\Delta t}$$

Note:
Making Connections: Force and Momentum
Force and momentum are intimately related. Force acting over time can change momentum, and Newton's second law of motion, can be stated in its most broadly applicable form in terms of momentum. Momentum continues to be a key concept in the study of atomic and subatomic particles in quantum mechanics.

This statement of Newton's second law of motion includes the more familiar $\mathbf{F}_{\text{net}}{=}m\mathbf{a}$ as a special case. We can derive this form as follows. First, note that the change in momentum $\Delta\mathbf{p}$ is given by
Equation:

$$\Delta \mathbf{p} = \Delta (m\mathbf{v}).$$

If the mass of the system is constant, then
Equation:

$$\Delta (m\mathbf{v}) = m\Delta \mathbf{v}.$$

So that for constant mass, Newton's second law of motion becomes
Equation:

$$\mathbf{F}_{\text{net}} = \frac{\Delta \mathbf{p}}{\Delta t} = \frac{m\Delta \mathbf{v}}{\Delta t}.$$

Because $\frac{\Delta \mathbf{v}}{\Delta t} = \mathbf{a}$, we get the familiar equation
Equation:

$$\mathbf{F}_{\text{net}}=m\mathbf{a}$$

when the mass of the system is constant.

Newton’s second law of motion stated in terms of momentum is more generally applicable because it can be applied to systems where the mass is changing, such as rockets, as well as to systems of constant mass. We will consider systems with varying mass in some detail**;** however, the relationship between momentum and force remains useful when mass is constant, such as in the following example.

Example:
Calculating Force: Venus Williams’ Racquet
During the 2007 French Open, Venus Williams hit the fastest recorded serve in a premier women’s match, reaching a speed of 58 m/s (209 km/h). What is the average force exerted on the 0.057-kg tennis ball by Venus Williams’ racquet, assuming that the ball’s speed just after impact is 58 m/s, that the initial horizontal component of the velocity before impact is negligible, and that the ball remained in contact with the racquet for 5.0 ms (milliseconds)?
Strategy
This problem involves only one dimension because the ball starts from having no horizontal velocity component before impact. Newton’s second law stated in terms of momentum is then written as
Equation:

$$\mathbf{F}_{\text{net}}=\frac{\Delta\mathbf{p}}{\Delta t}.$$

As noted above, when mass is constant, the change in momentum is given by
Equation:

$$\Delta p=m\Delta v=m(v_{\text{f}}-v_{\text{i}}).$$

In this example, the velocity just after impact and the change in time are given; thus, once Δp is calculated, $F_{\text{net}} = \frac{\Delta p}{\Delta t}$ can be used to find the force.

Solution

To determine the change in momentum, substitute the values for the initial and final velocities into the equation above.

Equation:

$$\begin{aligned}\Delta p &= m(v_{\text{f}} - v_{\text{i}}) \\ &= (0.057\ \text{kg})(58\ \text{m/s} - 0\ \text{m/s}) \\ &= 3.306\ \text{kg}\cdot\text{m/s} \approx 3.3\ \text{kg}\cdot\text{m/s}\end{aligned}$$

Now the magnitude of the net external force can determined by using $F_{\text{net}} = \frac{\Delta p}{\Delta t}$:

Equation:

$$\begin{aligned}F_{\text{net}} &= \frac{\Delta p}{\Delta t} = \frac{3.306\ \text{kg}\cdot\text{m/s}}{5.0\times10^{-3}\ \text{s}} \\ &= 661\ \text{N} \approx 660\ \text{N},\end{aligned}$$

where we have retained only two significant figures in the final step.

Discussion

This quantity was the average force exerted by Venus Williams' racquet on the tennis ball during its brief impact (note that the ball also experienced the 0.56-N force of gravity, but that force was not due to the racquet). This problem could also be solved by first finding the acceleration and then using $F_{\text{net}} = \text{ma}$, but one additional step would be required compared with the strategy used in this example.

Section Summary

- Linear momentum (*momentum* for brevity) is defined as the product of a system's mass multiplied by its velocity.
- In symbols, linear momentum $\mathbf{p}$ is defined to be
  Equation:

$$\mathbf{p} = m\mathbf{v},$$

where m is the mass of the system and $\mathbf{v}$ is its velocity.

- The SI unit for momentum is $\mathrm{kg \cdot m/s}$.
- Newton's second law of motion in terms of momentum states that the net external force equals the change in momentum of a system divided by the time over which it changes.
- In symbols, Newton's second law of motion is defined to be **Equation:**

$$\mathbf{F}_{\text{net}} = \frac{\Delta \mathbf{p}}{\Delta t},$$

$\mathbf{F}_{\text{net}}$ is the net external force, $\Delta \mathbf{p}$ is the change in momentum, and Δt is the change time.

Conceptual Questions

Exercise:

Problem:

An object that has a small mass and an object that has a large mass have the same momentum. Which object has the largest kinetic energy?

Exercise:

Problem:

An object that has a small mass and an object that has a large mass have the same kinetic energy. Which mass has the largest momentum?

Exercise:

Problem: Professional Application

Football coaches advise players to block, hit, and tackle with their feet on the ground rather than by leaping through the air. Using the

concepts of momentum, work, and energy, explain how a football player can be more effective with his feet on the ground.

Exercise:

Problem:

How can a small force impart the same momentum to an object as a large force?

Problems & Exercises

Exercise:

Problem:

(a) Calculate the momentum of a 2000-kg elephant charging a hunter at a speed of $7.50\ \text{m/s}$. (b) Compare the elephant's momentum with the momentum of a 0.0400-kg tranquilizer dart fired at a speed of $600\ \text{m/s}$. (c) What is the momentum of the 90.0-kg hunter running at $7.40\ \text{m/s}$ after missing the elephant?

Solution:

(a) $1.50 \times 10^{4}\ \text{kg} \cdot \text{m/s}$

(b) 625 to 1

(c) $6.66 \times 10^{2}\ \text{kg} \cdot \text{m/s}$

Exercise:

Problem:

(a) What is the mass of a large ship that has a momentum of $1.60 \times 10^{9}\ \text{kg} \cdot \text{m/s}$, when the ship is moving at a speed of $48.0\ \text{km/h}$? (b) Compare the ship's momentum to the momentum of a 1100-kg artillery shell fired at a speed of $1200\ \text{m/s}$.

Exercise:

Problem:

(a) At what speed would a 2.00×10^4-kg airplane have to fly to have a momentum of $1.60 \times 10^9\ \mathrm{kg \cdot m/s}$ (the same as the ship's momentum in the problem above)? (b) What is the plane's momentum when it is taking off at a speed of $60.0\ \mathrm{m/s}$? (c) If the ship is an aircraft carrier that launches these airplanes with a catapult, discuss the implications of your answer to (b) as it relates to recoil effects of the catapult on the ship.

Solution:

(a) $8.00 \times 10^4\ \mathrm{m/s}$

(b) $1.20 \times 10^6\ \mathrm{kg \cdot m/s}$

(c) Because the momentum of the airplane is 3 orders of magnitude smaller than of the ship, the ship will not recoil very much. The recoil would be $-0.0100\ \mathrm{m/s}$, which is probably not noticeable.

Exercise:

Problem:

(a) What is the momentum of a garbage truck that is 1.20×10^4 kg and is moving at $10.0\ \mathrm{m/s}$? (b) At what speed would an 8.00-kg trash can have the same momentum as the truck?

Exercise:

Problem:

A runaway train car that has a mass of 15,000 kg travels at a speed of $5.4\ \mathrm{m/s}$ down a track. Compute the time required for a force of 1500 N to bring the car to rest.

Solution:

54 s

Exercise:

Problem:

The mass of Earth is 5.972×10^{24} kg and its orbital radius is an average of 1.496×10^{11} m. Calculate its linear momentum.

Glossary

linear momentum
: the product of mass and velocity

second law of motion
: physical law that states that the net external force equals the change in momentum of a system divided by the time over which it changes

Introduction to Energy
Exercise:

UMASS AMHERST Instructor's Notes

Problem:

The rest of this chapter talks about the other important property of particles: Energy. These sections are straight from 131 and so should be familiar. They are included here for your reference. Feel free to skim them if you want.

So, what is energy? in short, energy is the ability to do work. That's the definition that we'll be using in this course. You can also flip this on its head with the change of perspective, however, and say that work is one way of transferring this quantity known as energy. Before we get too deep into it, it's probably worth comparing briefly the idea of energy and the idea forces. So, up to this point we've been talking about forces, and forces are an idea of instance. We look at the forces that are acting on an object "right now", and "right now" is all that matters, and then we can work instant by instant like we did in our simulations to study the motion of an object. Most physics courses begin with the idea of forces because they're easy to get a feel for what a force is., we've all experienced pushes and pulls.

Energy, on the other hand, is an example of a conservation law. Energy is a quantity that never changes through a process. This allows us to relate two points in time that are not directly next to each other. So, with forces, we had to take very tiny little time steps. With energy, we can go from the beginning of a process all the way to the end, not care too much about the middle, and know that energy is going to be conserved. So, why don't we start with energy? Well, energy can be conceptually a little bit more difficult to get your head around. What exactly is energy; it's a little bit more abstract of an idea, which is why most physics courses, including this one, put it off until after a discussion of forces.

So, what's the big point of the next three chapters? So, you've probably dealt with energy before in a previous science class. Our goal is to develop a coherent picture of energy across different scientific disciplines and across a large variety of different distant scales, from the sizes that we experience in our everyday world of people and cars and trees, all the way down to the atomic and molecular scale. We will, therefore, deal with several different ideas that can seem unrelated. We'll talk about boxes on hills and fish on springs and the kinetic energy of, say, cars, but we'll also talk about the kinetic energy of molecules, which we'll find out is directly related to the idea of temperature, and we'll talk about energy transfer through random collisions of particles on the atomic scale, which we'll talk about as the idea of heat. These different ideas and others including chemical bonds are connected by the idea of energy.

So, again, let's review how the next three chapters are laid out this current chapter provides a basic overview of what energy is, identifies the two main types of energy, kinetic and potential energy, identifies two main scales of energy we'll discuss in this class, the macroscopic scale of people, cars, etc., and the microscopic scale of atoms and molecules. The next chapter discusses energy on this macroscopic scale of people cars and so forth, and the chapter after that really gets into energy on the microscopic scale, atoms, molecules, and temperature.

Units of Energy
Exercise:

UMASS AMHERST Instructor's Notes

Problem:
Your Quiz will Cover

- Converting between the different units of energy

If energy is defined as the ability to do work, then energy and work must have the same units. Thus, the SI unit of the energy is the Joule (recall 1J=1Nm=1kgms2). Energy, however, is one quantity where there are many other units in common use in scientific literature including electron-Volts (eV), kilowatt-hours (kW·hr), calories, and Calories.
Exercise:

UMASS AMHERST Instructor's Notes

Problem:

In this course, we will be using Joules and electron-Volts exclusively. We are including these other units for your reference.

Electron-Volts

A common quantity in chemistry is the electron-Volt or eV. One electron-Volt is the amount of energy gained by an electron as it travels between the two ends of a 1 Volt battery (a concept that will be discussed in more detail when you study electricity). Numerically, $1eV = 1.602\times10^{-19}J$. The reason this unit is common in chemistry is that the energies of atomic bonds are

typically about 1eV as shown in the table below[footnote]. The bond-dissociation energy is the energy released when the bond is formed. "Bond-Dissociation Energy - Wikipedia." Accessed August 1, 2017. https://en.wikipedia.org/wiki/Bond-dissociation_energy.

Bond	**Bond-dissociation energy at 298K (eV/Bond)**	**Comment**
C-C	3.60-3.69	Strong, but weaker than C–H bonds
Cl-Cl	2.51	Indicated by the yellowish colour of this gas
H-H	4.52	Strong, nonpolarizable bondCleaved only by metals and by strong oxidants
O-H	4.77	Slightly stronger than C–H bonds
OH-H	2.78	Far weaker than C–H bonds
C-O	11.16	Far stronger than C–H bonds
O-CO	5.51	Slightly stronger than C–H bonds
O=O	5.15	Stronger than single bondsWeaker than many other double bonds

Bond	Bond-dissociation energy at 298K (eV/Bond)	Comment
N=N	9.79	One of the strongest bondsLarge activation energy in production of ammonia
H_3C-H	4.550	One of the strongest aliphatic C–H bonds

Exercise:

UMASS AMHERST Instructor's Notes

Problem:

Note that the eV is significantly smaller than the joule; eV will generally be the smallest unit of energy used in this course.

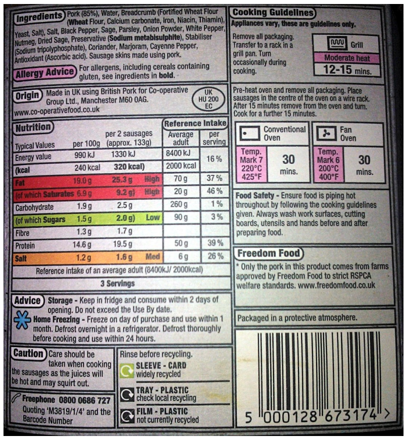
Kilowatt Hours

When you buy electricity from the power company, the bill says how many kilowatt hours you have purchased. A Watt is a unit of a quantity called *power* and 1 Watt is equal to 1 Joule/second: 1W = 1 J/s. Thus, a kilowatt hour is:

$$(1kW * hr)\left(\frac{1000W}{kW}\right)\left(\frac{1J/s}{W}\right)\left(\frac{3600s}{hr}\right) = 3.6106J = 3.6MJs$$

The calorie is an imperial unit of energy that is still in common use in the nutritional sciences in the United States. One calorie (lowercase c) is the amount of energy needed to raise 1g of water $1^{o}C$ or 1 cal = 4.814J. On food labels, you will see energy listed in Calories (capital C). One Calorie is

equal to 1kilocalorie; in other words, 1 Cal = 1000 cal. Thus, one 1 Cal = 4814 J. In other countries, you will see food labels in both Calories and Joules like the one shown in Figure 1.



A food label from the UK showing the energy of the food in both Joules and kcal (or Calories).

Types and Scales of Energy
Exercise:

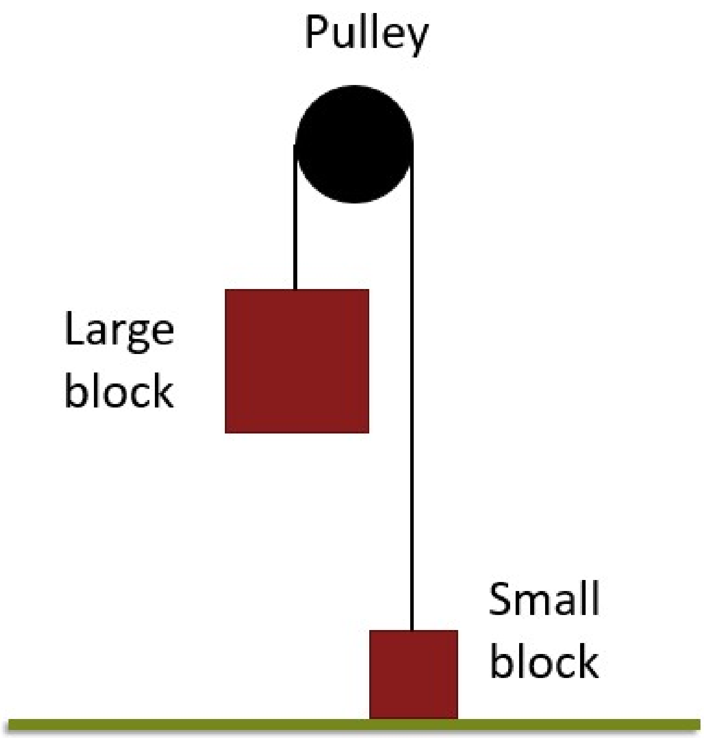
UMASS AMHERST Instructor's Notes

Problem:

This section provides the definitions of the terms kinetic and potential energy and defines the microscopic and macroscopic scales. Details and formulae will be provided in upcoming chapters.

Fundamentally, there are only two kinds of energy: kinetic energy and potential energy. *Kinetic energy (K) is the ability to do work associated with motion and potential energy (U) is the ability to do work arising from the relative positions of two or more objects.* As an example, the car in motion in the left image of Figure 2 has the capability to do work due to its motion - the car has *kinetic energy*. If the car were to crash, then a force would be exerted over a distance deforming the car (right image in Figure 2). The sheer fact that the car is moving means that it *can* do work. Similarly, the larger block in Figure 3 could do work if the system were released. As the large block fell, it would lift the small block. The large block has *potential energy* - an ability to do work due to its position relative to the earth.

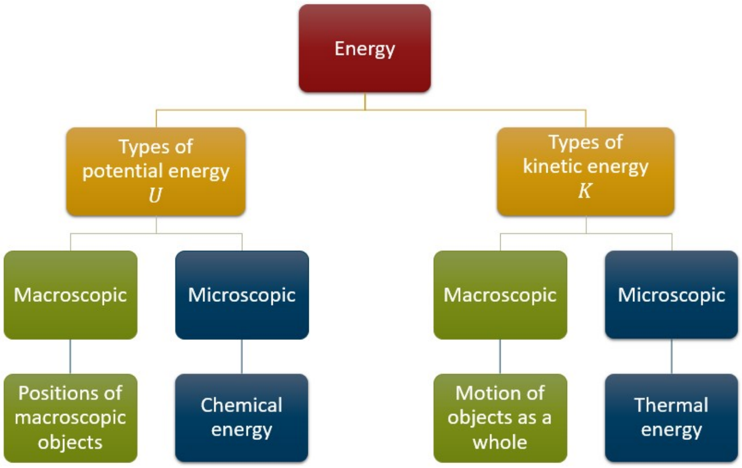
Figure B: A car traveling down the road (left) has an ability to do work due to its motion - it has kinetic energy. We see that ability to do work when the car crashes (right) - a force acts for a distance deforming the car.[footnote] SteveBaker. English: A 2007 MINI Cooper’S Car Shown Immediately before - and Soon after - a Severe Car Crash. At: Lat/Long 31.03669,-97.470881, February 9, 2009. Own work. https://commons.wikimedia.org/wiki/File:BeforeAndAfterMINICooperS.png.



A large block connected to a small block over a pulley has an ability to do work due to its position relative to the earth; the large block has potential energy. We see that work when the large block is released exerting, via the rope and pulley, a force on the small block for a distance causing it to accelerate upwards.

All of the many different types of energy that you have heard about in previous courses, thermal, chemical, electrical, etc., all ultimately boil down to these two different types. You may be wondering how chemical and thermal energy can be potential or kinetic. Typically when we think of kinetic energy, we think of the motion of people, cars, and the like! The key is to think about the *scale* of the energy: are we talking about energy at the macroscopic scale (people etc.) or the microscopic scale (atoms and molecules)? As we shall see, thermal energy is just kinetic energy on the microscopic scale and chemical energy is potential energy on the microscopic scale. The relationships between these types of energy can be

seen in Figure D. One of our goals throughout these chapters on energy is to develop a coherent picture of energy that applies at both the macroscopic scale of people etc. and at the microscopic scale of atoms and molecules. Thus, while we may present the macroscopic and microscopic scales in two separate chapters, keep in mind that we are talking about the same idea of energy throughout. At the end, we will look at how to transfer energy between these two different scales.



The relationships between different types of energy

Exercise:

UMASS AMHERST Instructor's Notes

Problem:

This chart provides a way to organize the different types of energy. We will flush out this chart in upcoming sections.

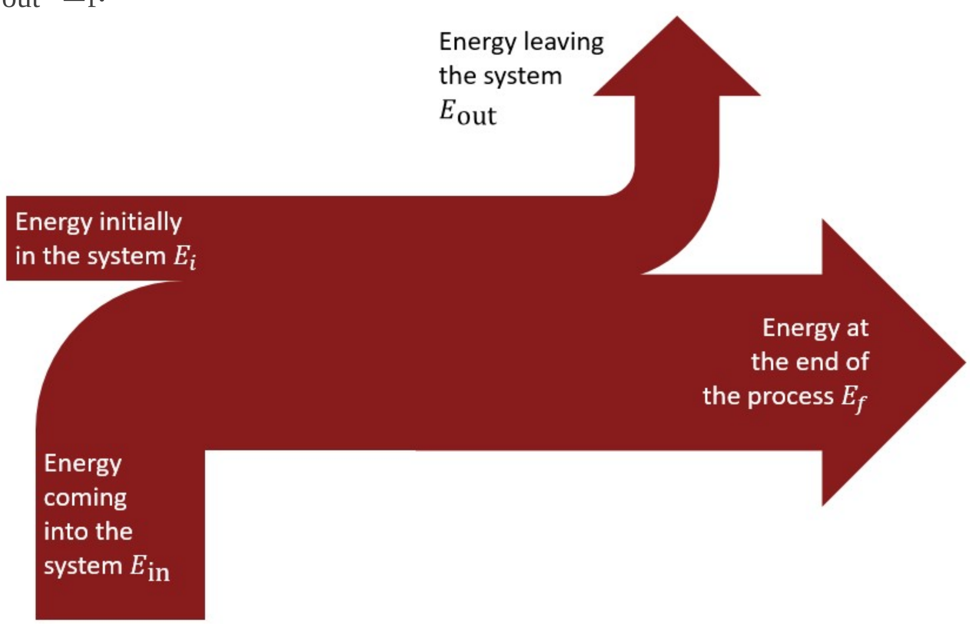
Conservation of Energy
Exercise:

UMASS AMHERST Instructor's Notes

Problem:
Your Quiz will Cover

- Understanding that, in a closed system, energy is conserved

You probably have heard from other courses that “energy is conserved.” This statement is true, for the Universe as a whole - the total amount of energy has not changed since the birth of the Universe 13.6Gyr ago. While this is a hugely important fact which gets deep into the heart of physics, it may not seem very useful. However, there is an, equally fundamental, and more useful fact: the amount of energy in any given system is conserved: the energy at the end of some process is what I had at the beginning plus any that came in minus any that went out, or in equation form: $E_i+E_{in}-E_{out}=E_f$.



The conservation of energy in graphical form $E_i+E_{in}-$

$$E_{out}=E_f$$

Ways to Transfer Energy
Exercise:

UMASS AMHERST Instructor's Notes

Problem:
Your Quiz will Cover

- Understanding the difference between heat and temperature
- Understanding the differences between the different types of heat transfer

So what ways are there to transfer energy into or out of a system? Well we already know of one way: work. If we do positive work on a system (the force we apply is in roughly the same direction as the displacement), then we will add energy *in*. Conversely, if we do negative work on a system (force essentially opposing the displacement) then energy is leaving the system. This should make some intuitive sense: we expect for positive work that the object will speed up - the object will gain kinetic energy. On the other hand, if we do negative work, we expect that the object will lose kinetic energy.

There is another way to transfer energy into or out of a system: heat which we represent by the letter Q. At its core, *heat is the transfer of energy by collisions at the microscopic scale*.

Heat

We already know that work is one way in which we can transfer energy, but you might be thinking to yourself, is there another way we can transfer energy into or out of a system? We know, for example, that if I place a warm soda in contact with the cold ice cube, the soda cools and some of the ice cube melts until the two come to a constant temperature. It certainly

seems that energy is being transferred in this process; how is the energy being transferring in a process like this?

This method of energy transfer is known as heat. Heat is a transfer of energy; an object cannot have heat any more than an object can have work. It's also important to keep in mind the distinction between heat and temperature. The symbol for heat that we will use is the letter Q. Since it is a measure of transferred energy, heat will have the unit of joules, just like work and energy do. The sign convention for heat that we will adopt is that when energy flows into a system, Q is positive, and when energy flows out of a system, heat is negative.

There are three main ways to transfer heat into or out of a system. The first method is conduction, which is essentially when you place a metal stick in a fire, the other end of the stick gets hot, the second method is convection, which occurs when you have say a pot of water over a flame, and the third method is radiation, where light from the Sun warms the earth. Let's now explore these three processes in a little bit more detail.

Conduction

Conduction occurs when atoms in a material collide with each other. So. let's assume we have some sort of solid, and of course inside the solid the atoms are arranged in some regular pattern. When we place one end the solid in the flame, the atoms nearest the flame gain energy and begin to vibrate more vigorously, increasing their temperature. These atoms bump into the atoms next to them, which bump into the atoms next for them, and so on and so on, transferring energy up the rod. This transfer of energy through the molecular collisions of the atoms within the rod is known as heat. In a solid, the atoms are very close together and can easily bump into each other, and therefore solids transfer heat quite well. In liquids and gases, on the other hand, the atoms are a little further apart, and so don't collide as often. They can still transfer energy as heat through conduction, just not as well, because the collisions won't be as frequent. in a metal on the other hand, the C electrons that bind the metal together are free to move around and collide with each other. These C electrons, since they can travel

so far, result in a lot of collisions, and result in metals conducting heat very well. Heat will quickly result in the transfer of energy from one end of a piece of metal to the other, which is why it’s easy to burn your hand on a piece of metal that’s hot on one end.

Radiation

So, in our first example, the fire caused the atoms nearest to it in the solid to begin to move. Well, how did this happen? Well the fire generates particles of light energy called photons that will be discussed in some detail in Physics 132. These photons, some of them travel and collide with the atoms in the solid, giving them their energy and causing them to move, and setting off the chain that we talked about in conduction. This transfer of energy through molecular collisions involving photons is called heat by radiation. This is the mechanism by which the earth gets its energy from the Sun. There is no matter in between the earth and the Sun to provide a mechanism for conduction. The only mechanism of heat transfer is through radiation of photons emitted by the Sun, which can travel through the vacuum of space.

Convection

The final way the transfer heat is called convection. Now, convection is a little bit more complicated than the other mechanisms, so we won’t go into it in detail, but in short, convection is the transfer of energy through the bulk motion of a fluid, so a fluid in physics means either a liquid or a gas, and by the bulk motion, we mean giant currents of liquid or gas moving through the material. In our example of a pot of water on a stove, the liquid near the bottom of the pot gains energy through conduction. Flame gets the bottom of the pot warm through radiation, and the bottom of the pot is in contact with the liquid and transfers energy by conduction. Now the water at the bottom of the pot is warmer than the surrounding. Warmer fluids tend to have lower density, so this warm fluid near the bottom rises to the top carrying its energy with it, and cooler liquid settles to the bottom, and thus we get this current up in the middle and down the outside, bulk motion of

the water transferring energy through the system. This is something that we see a lot and is of critical importance in the motion of the oceans. Cool water from the poles sinks to the bottom of the ocean, travels down to the equator, where it’s warmed up by the Sun, and then rises back to the top, distributing energy and nutrients throughout the ocean.

Summary

So, now we’ve talked about the three methods of heat transfer, conduction, radiation, and convection. What’s the commonality between these three different methods? These three different methods are, at the microscopic level, the transfer of energy through random collisions. In conduction, the atoms are colliding with each other, in radiation you have the collision of photons with atoms, and in convection you have the transfer of energy through the motion of the fluid itself. And this is what heat really is, it’s the transfer of energy through collisions at the microscopic scale.

Heat, which we will use the letter Q to represent, is another way of transferring energy. Energy entering the system has positive heat in our convention, and energy leaving the system has negative heat in our convention. This energy transfer can occur in one of three ways, conduction, convection, or radiation, but these methods are, at the molecular level, collisions between particles.

Exercise:

UMASS AMHERST Instructor's Notes

Problem:(insert text here)

This paragraph summarizes things nicely and is important for your quiz! You need to know what heat is and how it is different from temperature. You also need to know that if energy is going into a system, then we count it as positive heat or work. Energy leaving is negative heat or work.

The Formal Statement of the Conservation of Energy as the First Law of Thermodynamics
Exercise:

UMASS AMHERST Instructor's Notes

Problem:

While this section is more formally mathematical than most, you should pay attention as this is probably the most important idea in all of science! For your quiz, you will need to be able to use this idea in the abstract. If I give you two of ΔE, Q, or W you should be able to tell me the third.

In section 13.4, we stated that energy must be conserved: $E_i + E_{in} - E_{out} = E_f$. Moving things around we get $E_f - E_i = E_{in} - E_{out}$. Recognizing the term on the left as E, we can say $E = E_{in} - E_{out}$. If we redefine E_{in} and E_{out} as just different directions of transferred energy $E_{transferred}$, then we have $E = E_{transferred}$ where $E_{transferred}$ is positive if energy comes into the system and negative if energy is leaving the system. Now, we know that there are two different ways to transfer energy into or out of a system: heat Q and work W. Thus, $E_{transferred}$ must be the sum of the energy transferred by heat and the energy transferred by work, $E_{transferred} = Q + W$. The statement of the law of conservation of energy can therefore be written as

Written in this form, the law of conservation of energy is called the *First Law of Thermodynamics*, i.e. the First Law of Thermodynamics and the Law of Conservation of Energy are the same thing.

This statement is so fundamental to the idea of physics that it is worth spending a minute to really unpack what it says. Looking again at the First Law of Thermodynamics (with the delta expanded) we see

where both heat Q and work W are ways of *transferring* energy into or out of the system. As a first example, say we have some system and we do work on that system without transferring any energy as heat. In this case, $W>0$ as energy is coming in and $Q=0$. The result is that $E_f>E_i$, which makes sense as we have added energy. Similarly, if we had a system that is losing heat to its environment while remaining stationary at constant volume then we know that $Q<0$ because heat is flowing out and $W=0$ due to the fact that there is no “distance” for $W=Fd\cos\theta$. Therefore, $Q+W<0$ and $E_f<E_i$ as expected given that energy is flowing out of the system.

Why the First Law of Thermodynamics May Look Different in Your Other Courses

Exercise:

UMASS AMHERST Instructor's Notes

Problem:

There are some differences in conventions when expressing the first law of thermodynamics to be aware of.

In some other courses or references, you may see the first law of thermodynamics written as $\Delta E = Q - W$, i.e. the sign of work may be different. This is still the same First Law of Thermodynamics/Law of Conservation of Energy that we are talking about here. The difference is one of perspective. In this class, we are considering energy flowing into the system as positive and energy flowing out of the system as negative. This convention matches our convention for heat Q as well as matching our definition of work from mechanics $W = Fdcos\theta$ which considers only external forces. Physically, we are thinking about work done *on* the system by *external* forces.

To understand the $\Delta E = Q - W$ formulation, you need a bit of history. The Laws of Thermodynamics were formulated during the Industrial Revolution as people were studying the properties of steam engines and the like. When studying the performance of a steam engine, the interesting quantity is not the work done *on the system by external* forces, but instead the work done *by the engine on its environment*. Stated another way, the developers of the Laws of Thermodynamics were not using our idea of object egoism! Instead of thinking about $\overrightarrow{F}_{environment \rightarrow engine}$, they were thinking about $\overrightarrow{F}_{engine \rightarrow environment}$. Now by Newton's Third Law, these two forces are equal except for a negative sign. Thus, when you think about work done *by* the engine instead of the work done *on* the system, work flips sign and you end up with $\Delta E = Q - W$ instead of $\Delta E = Q + W$.

In this class, we will stick with $\Delta E = Q + W$, i.e. we will use the same definition for work we have been using. The takeaway from this section is that you may see the First Law of Thermodynamics written with a different sign for work. Different fields use different conventions (it would be nice if we could agree, but oh well). Therefore, you should be aware that writing it as $\Delta E = Q - W$ is just a different perspective born out of the historical development of science. This quirk with the sign of work is a great example of the impact that history and all of its associated socio-economic factors can have on the history of science. One wonders what other ideas could be expressed more coherently? What scientific questions have not been explored because the people in power doing the research did not value them?

Introduction to Energy of Objects as a Whole on the Macroscopic Scale

For starters, it is simpler to think about energy of objects as a whole at the macroscopic scale separate from the collective energy of the constituent molecules. This thinking is in line with the physics problem solving approach of starting simple and adding complications later. The next chapter deals with energy at the microscopic realm. We will get into connecting these two realms in class. We shall see that there are only specific ways of transferring energy between the macroscopic world and the microscopic world so separating these two regimes makes sense. As you know from the previous chapter, heat is the transfer of energy by microscopic collisions. *Thus, heat is really only important at the microscopic scale and will not be considered in this chapter.*

Kinetic Energy of an Object

- Explain work as a transfer of energy and net work as the work done by the net force.
- Explain and apply the work-energy theorem.

Exercise:

UMASS AMHERST Instructor's Notes

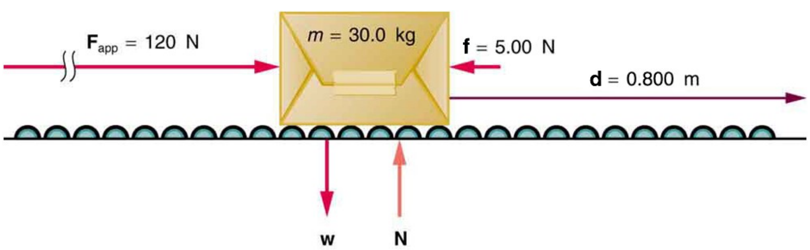
Problem:

Our goal in this section is to figure out an expression for the kinetic energy. While you should try to understand where the expression for kinetic energy comes from as it will help you understand how energy and work are related, the key is the expression we get to at the end.

Our goal in this section is to figure out an expression for the kinetic energy.

Figuring Out the Expression for Kinetic Energy

To achieve this objective, let's begin our study of energy with, as usual the simplest possible situation. Consider a one-dimensional situation where a force is used to accelerate an object in a direction parallel to its initial velocity. Such a situation occurs for the package on the roller belt conveyor system shown in Figure 1.



A package on a roller belt is pushed horizontally through a distance d

In this case, there is no transfer of energy by molecular collisions, i.e. there is no heat and Q=0. Meaning that our statement of conservation of energy goes from

$$E = Q + W$$

to

$$E = W$$

Similarly, there is no ability to do work due to position; the box cannot fall because of the rollers. Thus, there is no potential energy in this problem and all of our energy is kinetic energy: E=K. Therefore, our statement of conservation of energy for this situation is just

$$\Delta K = W$$

$$K_f - K_i = W$$

The effect of the net force Fnet is to accelerate the package from v_0 to v. The kinetic energy of the package increases, indicating that the net work done on the system is positive. (SeeExample.) By using Newton's second law, and doing some algebra, we can reach an expression for kinetic energy.

The force of gravity and the normal force acting on the package are perpendicular to the displacement and do no work. Moreover, they are also equal in magnitude and opposite in direction so they cancel in calculating the net force. The net force arises solely from the horizontal applied force Fapp and the horizontal friction force f. Thus, as expected, the net force is parallel to the displacement, so that $\theta=0^o$ and $\cos\theta=1$, and the net work is given by

*W*net=*F*net*d*.

Substituting *F*net=*ma* from Newton's second law gives

$$Wnet = mad$$

To get a relationship between net work and the speed given to a system by the net force acting on it, we take

$$d = x - x_0$$

and use the equation studied inMotion Equations for Constant Acceleration in One Dimension for the change in speed over a distance *d* if the acceleration has the constant value *a*; namely, $v^2=v_0{}^2+2ad$ (note that *a* appears in the expression for the net work). Solving for acceleration gives $a=(v^2-v02)/2d$. When *a* is substituted into the preceding expression for W_{net}, we obtain

$$W_{net} = m(\frac{v^2 - v_0^2}{2d})d$$

The *d* cancels, and we rearrange this to obtain

$$W_{net} = \frac{1}{2}mv^2 - \frac{1}{2}mv_0^2$$

Exercise:

UMASS AMHERST Instructor's Notes

Problem:

The following subsection is based on
umdberg / Kinetic energy and the work-energy theorem (2013). Available at: http://umdberg.pbworks.com/w/page/68405433/Kinetic%20energy%20and%20the%20work-energy%20theorem%20(2013). (Accessed: 9th August 2017)

Interpreting the Result: Kinetic Energy

Exercise:

UMASS AMHERST Instructor's Notes

Problem:

While, this section is short, it is arguably more important than the text above. This section is about interpreting our expression for kinetic energy and helping you understand how it is different from the, at first glance, similar looking momentum.

This section is based upon http://umdberg.pbworks.com/w/page/68405433/Kinetic%20energy%20and%20the%20work-energy%20theorem%20(2013)

What has come out after all our manipulations is a that the work in this case is related to a change in a quantity associated with motion, $1/2mv^2$. This is kind of like momentum in that it counts both the mass and the velocity, but it differs in that momentum is proportional to the velocity vector -- so it is very directional. Reversing momentum is a big deal even if the speed doesn't change. For our new quantity, since it is proportional to v^2 instead of to v, the direction of motion doesn't matter. You get the same v^2 whether v is positive or negative. If our general result turns out to only depend on the magnitude of v and not the direction (it will), we will have solved our problem and learned what it is that changes an object's speed (not caring about direction).

When you compare the result of our manipulations to our analysis in terms of energy, you can see that $1/2mv^2$ must be the *kinetic energy*. It is a measure of "the energy associated with how much an object is moving".

Examples Applying Conservation of Energy with only Kinetic Energy
Exercise:

UMASS AMHERST Instructor's Notes

Problem:

For your quiz, you are expected to be able to solve problems such as these.

Example:
Calculating the Kinetic Energy of a Package
Suppose a 30.0-kg package on the roller belt conveyor system in [link] is moving at 0.500 m/s. What is its kinetic energy?
Strategy
Because the mass m and speed v are given, the kinetic energy can be calculated from its definition as given in the equation $\mathrm{KE}=\frac{1}{2}mv^2$.
Solution
The kinetic energy is given by
Equation:

$$\mathrm{KE}=\frac{1}{2}mv^2.$$

Entering known values gives
Equation:

$$\mathrm{KE}=0.5(30.0\ \mathrm{kg})(0.500\ \mathrm{m/s})^2,$$

which yields
Equation:

$$\mathrm{KE}=3.75\ \mathrm{kg}\cdot\mathrm{m}^2/\mathrm{s}^2=3.75\ \mathrm{J}.$$

Discussion
Note that the unit of kinetic energy is the joule, the same as the unit of work, as mentioned when work was first defined. It is also interesting that, although this is a fairly massive package, its kinetic energy is not large at this relatively low speed. This fact is consistent with the observation that people can move packages like this without exhausting themselves.

Example:
Kinetic Energy of a Car
A car travels at 5m/s when it accelerates to 10m/s. After the car has finished accelerating, by what factor did its kinetic energy increase?
Solution
We are interested in the ratio of the final to the initial kinetic energies

$$\frac{K_f}{K_i}$$

Substituting our definition for kinetic energy we get

$$\frac{K_f}{K_i} = \frac{\frac{1}{2}m{v_f}^2}{\frac{1}{2}m{v_i}^2}$$

Since the mass of the car does not change, m and the $\frac{1}{2}$ cancel leaving

$$\frac{K_f}{K_i} = \frac{{v_f}^2}{{v_i}^2}$$

Substituting our values in we get

$$\frac{K_f}{K_i} = \frac{10^2}{5^2} = \frac{100}{25} = 4$$

Analysis
The speed went up by two, but the kinetic energy went up by a factor of four, a result consistent with the fact that kinetic energy depends upon the

square of the velocity. Speed matters a lot when thinking about energy!

Example:
Deep Space 1
Deep Space 1 was a space probe, launched on October 24, 1998, and it used a type of engine called a ion propulsion drive. This engine generates a weak force, but it can do so over a long period of time and using only a small amount of fuel. The probe has a mass of 474 kg and is traveling with an initial speed of 275 m/s. The only force acting on the probe is from the ion drive, at a force of 0.056N parallel to the probe's displacement, which is 2.42 million km. What is the final speed of the probe?
Solution
In this problem, we're looking for a change in speed, which tells us that we're looking for a change in the kinetic energy. With the only force on the probe being the ion drive, we can use the work-energy theorem to find the change in energy:

$$W = \mathit{\Delta} E$$

Then, we can expand the energy:

$$W = E_f - E_i$$

Since the only energy in this system is the kinetic energy, we can substitute the energies with our definition of kinetic energy:

$$W = \frac{1}{2}mv_f^2 - \frac{1}{2}mv_i^2$$

Since it's the final velocity that we're looking for, we can solve this equation for the final velocity:

$$v_f = \sqrt{\frac{W + \frac{1}{2}mv_i^2}{\frac{1}{2}m}} = \sqrt{\frac{2W}{m} + v_i^2}$$

The mass and the initial velocity are both given, but we need to find the work done on the probe. We can use our definition of work to calculate it:

$$W = F\Delta x = 0.056N * 2.42x10^9 m = 135520000J$$

Substituting our values and calculating the final velocity:

$$v_f = \sqrt{\frac{2(135520000J)}{474kg} + (275m/s)^2} = 804m/s$$

Macroscopic Potential Energy

- Explain gravitational potential energy in terms of work done against gravity.
- Show that the gravitational potential energy of an object of mass m at height h on Earth is given by $\text{PE}_\text{g} = \text{mgh}$.
- Show how knowledge of the potential energy as a function of position can be used to simplify calculations and explain physical phenomena.

Work Done Against Gravity

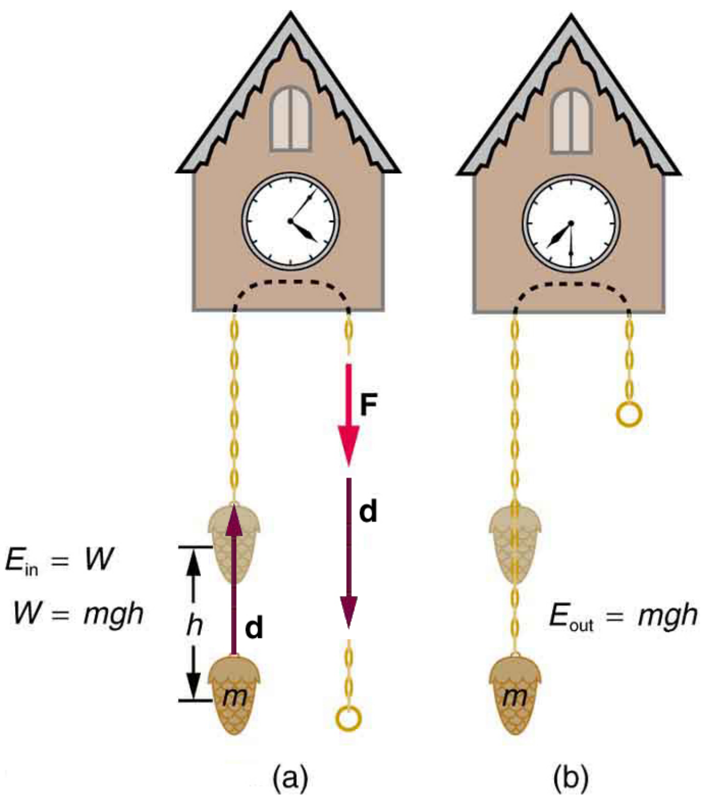
Climbing stairs and lifting objects is work in both the scientific and everyday sense—it is work done against the gravitational force. When there is work, there is a transformation of energy. The work done against the gravitational force goes into an important form of stored energy that we will explore in this section.

Let us calculate the work done in lifting an object of mass m through a height h, such as in [link]. If the object is lifted straight up at constant speed, then the force needed to lift it is equal to its weight mg. The work done on the mass is then $\text{W} = \text{Fd} = \text{mgh}$. We define this to be the **gravitational potential energy** (U_g) put into (or gained by) the object-Earth system. This energy is associated with the state of separation between two objects that attract each other by the gravitational force. For convenience, we refer to this as the U_g gained by the object, recognizing that this is energy stored in the gravitational field of Earth. Why do we use the word “system”? Potential energy is a property of a system rather than of a single object—due to its physical position. An object’s gravitational potential is due to its position relative to the surroundings within the Earth-object system. The force applied to the object is an external force, from outside the system. When it does positive work it increases the gravitational potential energy of the system. Because gravitational potential energy depends on relative position, we need a reference level at which to set the potential energy equal to 0. We usually choose this point to be Earth’s surface, but this point is arbitrary; what is important is the *difference* in gravitational potential energy, because this difference is what relates to the work done. The difference in gravitational potential energy of an object (in

the Earth-object system) between two rungs of a ladder will be the same for the first two rungs as for the last two rungs.

Converting Between Potential Energy and Kinetic Energy

Gravitational potential energy may be converted to other forms of energy, such as kinetic energy. If we release the mass, gravitational force will do an amount of work equal to mgh on it, thereby increasing its kinetic energy by that same amount (by the work-energy theorem). We will find it more useful to consider just the conversion of U_g to KE without explicitly considering the intermediate step of work. (See [link].) This shortcut makes it is easier to solve problems using energy (if possible) rather than explicitly using forces.



(a) The work done to lift the weight is stored in the mass-Earth system as gravitational potential energy. (b) As

the weight moves downward, this gravitational potential energy is transferred to the cuckoo clock.

More precisely, we define the *change* in gravitational potential energy $\Delta \mathrm{U_g}$ to be

Equation:

$$\Delta \mathrm{U_g} = \mathrm{mgh},$$

where, for simplicity, we denote the change in height by h rather than the usual Δh. Note that h is positive when the final height is greater than the initial height, and vice versa. For example, if a 0.500-kg mass hung from a cuckoo clock is raised 1.00 m, then its change in gravitational potential energy is

Equation:

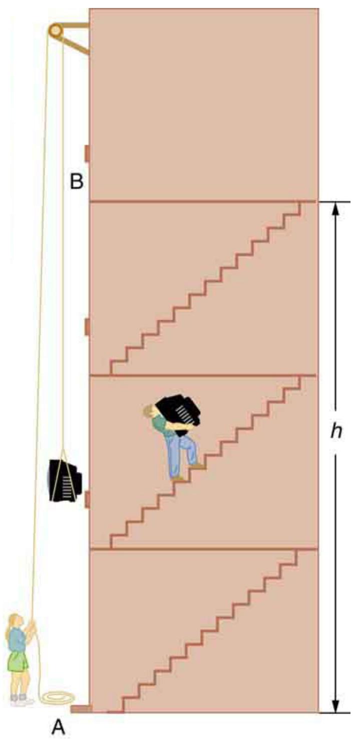
$$\begin{array}{lll} mgh & = & (0.500\ \mathrm{kg})\left(9.80\ \mathrm{m/s^2}\right)(1.00\ \mathrm{m}) \\ & = & 4.90\ \mathrm{kg \cdot m^2/s^2} = 4.90\ \mathrm{J}. \end{array}$$

Note that the units of gravitational potential energy turn out to be joules, the same as for work and other forms of energy. As the clock runs, the mass is lowered. We can think of the mass as gradually giving up its 4.90 J of gravitational potential energy, *without directly considering the force of gravity that does the work.*

Using Potential Energy to Simplify Calculations

The equation $\Delta \mathrm{U_g} = \mathrm{mgh}$ applies for any path that has a change in height of h, not just when the mass is lifted straight up. (See [link].) It is much easier to calculate mgh (a simple multiplication) than it is to calculate the work done along a complicated path. The idea of gravitational potential

energy has the double advantage that it is very broadly applicable and it makes calculations easier. From now on, we will consider that any change in vertical position h of a mass m is accompanied by a change in gravitational potential energy mgh, and we will avoid the equivalent but more difficult task of calculating work done by or against the gravitational force.



The change in gravitational potential energy $(\Delta \mathrm{U_g})$ between points A and B is independent of the path. $\Delta \mathrm{U_g} = \mathrm{mgh}$ for any path between the two points. Gravity

is one of a small class of forces where the work done by or against the force depends only on the starting and ending points, not on the path between them.

Example:
The Force to Stop Falling
A 60.0-kg person jumps onto the floor from a height of 3.00 m. If he lands stiffly (with his knee joints compressing by 0.500 cm), calculate the force on the knee joints.
Strategy
This person's energy is brought to zero in this situation by the work done on him by the floor as he stops. The initial $\mathrm{U_g}$ is transformed into KE as he falls. The work done by the floor reduces this kinetic energy to zero.
Solution
The work done on the person by the floor as he stops is given by
Equation:

$$W = \mathrm{Fd} \cos \theta = -\mathrm{Fd},$$

with a minus sign because the displacement while stopping and the force from floor are in opposite directions $(\cos \theta = \cos 180^{\circ} = -1)$. The floor removes energy from the system, so it does negative work.
The kinetic energy the person has upon reaching the floor is the amount of potential energy lost by falling through height h:
Equation:

$$\mathrm{KE} = -\Delta \mathrm{U_g} = -\mathrm{mgh},$$

The distance d that the person's knees bend is much smaller than the height h of the fall, so the additional change in gravitational potential energy during the knee bend is ignored.
The work W done by the floor on the person stops the person and brings the person's kinetic energy to zero:
Equation:

$$W = -\mathrm{KE} = \mathrm{mgh}.$$

Combining this equation with the expression for W gives
Equation:

$$-\mathrm{Fd} = \mathrm{mgh}.$$

Recalling that h is negative because the person fell *down*, the force on the knee joints is given by
Equation:

$$F = -\frac{\mathrm{mgh}}{d} = -\frac{(60.0\ \mathrm{kg})\left(9.80\ \mathrm{m/s}^2\right)(-3.00\ \mathrm{m})}{5.00\times 10^{-3}\ \mathrm{m}} = 3.53\times 10^{5}\ \mathrm{N}.$$

Discussion
Such a large force (500 times more than the person's weight) over the short impact time is enough to break bones. A much better way to cushion the shock is by bending the legs or rolling on the ground, increasing the time over which the force acts. A bending motion of 0.5 m this way yields a force 100 times smaller than in the example. A kangaroo's hopping shows this method in action. The kangaroo is the only large animal to use hopping for locomotion, but the shock in hopping is cushioned by the bending of its hind legs in each jump.(See [link].)

The work done by the ground upon the kangaroo reduces its kinetic energy to zero as it lands. However, by applying the force of the ground on the hind legs over a longer distance, the impact on the bones is reduced. (credit: Chris Samuel, Flickr)

We have seen that work done by or against the gravitational force depends only on the starting and ending points, and not on the path between, allowing us to define the simplifying concept of gravitational potential energy. We can do the same thing for a few other forces, and we will see that this leads to a formal definition of the law of conservation of energy.

The Zero of Potential Energy

This is perhaps a slightly subtler topic than you might first imagine. Let's think about this quantity of gravitational potential energy written as mgh. Here we have Mr. clumsy dropping a ball.

Mr. Clumsy dropping the ball.

Where is the gravitational potential energy of the ball equal to zero? Well, m and g are both numbers, m is the mass of the ball and g is 9.8 m/s^2, so essentially this is the same question as where is height equal to 0, h. Well, the logical place you might think for h to be equal to 0 would be the ground. We define the ground to be h=0, then the gravitational potential energy on the ground, U_g, is going to be zero when the ball is on the ground. Now let's move Mr. Clumsy to a platform on the top of a skyscraper.

Mr. Clumsy dropping the ball on top of a skyscraper, with a subway tunnel below.

Now where is h=0? This is a perhaps a little bit trickier question; do we define h=0 to be at the platform, or do we define it to be in the ground, or do we even define it to be in the subway tunnel underneath the skyscraper? Which of these should we choose for h=0, or which of these should we choose for the zero of gravitational potential energy? Well, the universe doesn't care where we choose h to be equal to 0. so due to that, any of these

choices, the platform, the ground, the subway tunnel, they're all fine, doesn't matter which we pick. You should just be very explicit with your choice, so when you're approaching a problem with gravitational potential energy, explicitly write down that I am going to choose the zero of gravitational potential energy to be the platform, for example. If I choose the ground, say, to have zero potential gravitational potential energy, then points in the subway tunnel below the ground have negative gravitational potential energy. There's absolutely nothing wrong with that.

Why is there nothing wrong with negative potential energy? Well, the work done by gravity is equal to the negative of the change in gravitational potential energy, or

$$W = -\Delta U$$

ΔU is always U_f-U_i, so the work done is

$$W = U_i - U_f$$

or using mgh for potential energy

$$W = mgh_i - mgh_f$$

The key point in all of this is that the work done does not depend upon the value of potential energy itself, only the change in the potential energy is a relevant quantity, and the change in potential.

Potential Energy of a Spring

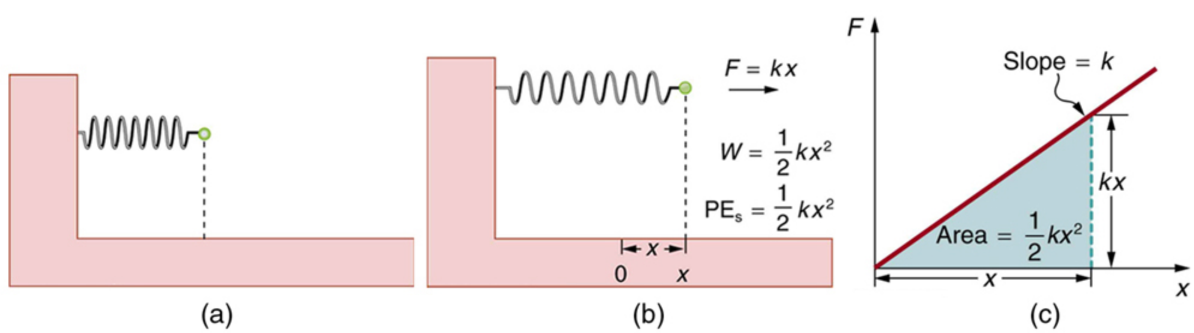
First, let us obtain an expression for the potential energy stored in a spring (U_s). We calculate the work done to stretch or compress a spring that obeys Hooke's law. (Hooke's law was examined in Elasticity: Stress and Strain, and states that the magnitude of force F on the spring and the resulting deformation ΔL are proportional, $F = k\Delta L$.) (See [link].) For our spring, we will replace ΔL (the amount of deformation produced by a force F) by the distance x that the spring is stretched or compressed along its length. So the force needed to stretch the spring has magnitude $\mathrm{F} = \mathrm{kx}$, where k is the spring's force constant. The force increases linearly from 0 at the start to kx

in the fully stretched position. The average force is $kx/2$. Thus the work done in stretching or compressing the spring is $W_s = \mathrm{Fd} = \left(\frac{kx}{2}\right)x = \frac{1}{2}kx^2$. Alternatively, we noted in Kinetic Energy and the Work-Energy Theorem that the area under a graph of F vs. x is the work done by the force. In [link](c) we see that this area is also $\frac{1}{2}kx^2$. We therefore define the **potential energy of a spring**, $\mathrm{U_s}$, to be

Equation:

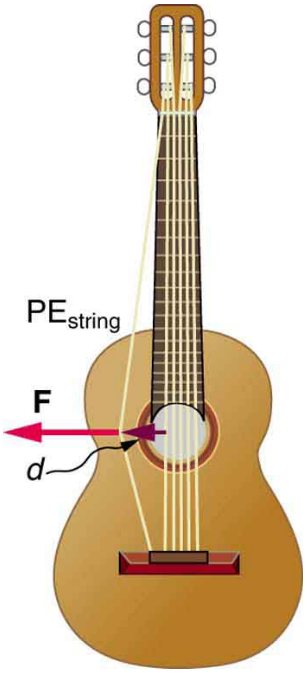
$$\mathrm{U_s} = \frac{1}{2}\mathrm{kx}^2,$$

where k is the spring's force constant and x is the displacement from its undeformed position. The potential energy represents the work done *on* the spring and the energy stored in it as a result of stretching or compressing it a distance x. The potential energy of the spring $\mathrm{U_s}$ does not depend on the path taken; it depends only on the stretch or squeeze x in the final configuration.



(a) An undeformed spring has no $\mathrm{U_s}$ stored in it. (b) The force needed to stretch (or compress) the spring a distance x has a magnitude $F = kx$, and the work done to stretch (or compress) it is $\frac{1}{2}kx^2$. Because the force is conservative, this work is stored as potential energy $(\mathrm{U_s})$ in the spring, and it can be fully recovered. (c) A graph of F vs. x has a slope of k, and the area under the graph is $\frac{1}{2}kx^2$. Thus the work done or potential energy stored is $\frac{1}{2}kx^2$.

The equation $U_s = \frac{1}{2}kx^2$ has general validity beyond the special case for which it was derived. Potential energy can be stored in any elastic medium by deforming it. Indeed, the general definition of **potential energy** is energy due to position, shape, or configuration. For shape or position deformations, stored energy is $U_s = \frac{1}{2}kx^2$, where k is the force constant of the particular system and x is its deformation. Another example is seen in [link] for a guitar string.



Work is done to deform the guitar string, giving it potential energy. When released, the potential energy is

converted to kinetic energy and back to potential as the string oscillates back and forth. A very small fraction is dissipated as sound energy, slowly removing energy from the string.

Glossary

gravitational potential energy
: the energy an object has due to its position in a gravitational field

Conservative vs Non-Conservative Forces

Let’s begin by considering a ball near the surface of the earth. The ball begins three meters above the ground, falls, and bounces back up to its original height. How much work is done by gravity in this case? Well, work is always defined as the force of interest times the displacement times the cosine of the angle between the force and the displacement. In this case the force we’re interested in is gravity, mg, so the work done is $mgdcos(\theta)$. On the way down, the force is parallel to the displacement. The ball is moving down and the force is down, so the angle between the force and the displacement is zero degrees. The cosine of zero degrees is 1, so the work done by gravity as the ball travels down is positive mgd on the way up. However, the force is down and the displacement is up, so the force and the displacement are antiparallel, or the angle between them is 180 degrees. Now the cosine of 180 degrees is -1, and so the work done by gravity as the ball travels back up is $-mgd$. The total work done by gravity on the entire loop is the sum of the work done by gravity on the way down plus the work done by gravity on the way back up, which in this case is equal to zero.

The ball in this example is moving on what is called a closed path. The ball starts and stops in the same place. We’ve just seen that for this closed path, the work done by the gravitational force is equal to zero. It turns out that the work done by gravity on any closed path is equal to zero. This leads to a question: is this statement true for all forces? Is the work done by any force on any closed path always equal to zero?

To answer this question, let’s think about a different force: friction. In this example, we have a 2kg box as it’s dragged three meters across a table, with a coefficient of kinetic friction of 0.2, and back, and we’re interested in the work done by the force of kinetic friction over this closed path. Once again, the work as always is the force times the displacement times the cosine of the angle in between. In this case, the force of interest is the force of kinetic friction, which we know to write as the coefficient of kinetic friction, μk, times the normal force. The box is not moving in the vertical direction, which I’ve called y in this example, so we know that the acceleration in the y direction is equal to zero. Thus, by Newton’s second law, we know that the force in the y direction is equal to zero, the net force is equal to zero.

Thus, we can conclude that the normal force in this problem, is equal to the weight of the box, mg. Thus, we have the force of kinetic friction, $\mu k * mg$. As the box is dragged to the left, the force of friction is opposite the displacement, thus the angle between them is 180 degrees. The box is moving to the left, but the force is opposing the motion, the angle between them is 180 degrees. The cosine of 180 degrees is, once again, -1. And so, the work done by the force of friction as it moves to the left is $-\mu k(mgd)$. On the way to the right, the force is still opposite the displacement. Now the block is going to the right, but the friction is still opposing the motion, so the angle between the force and the displacement is still 180 degrees, which means that the work done by friction as the block goes to the right is also $-\mu k(mgd)$. Thus, we see in this case that the work done by the force of friction around this closed path, the box starts and stops in the same place, so this is a closed path, the work done on this closed path is $-2\mu k(mgd)$, which is not equal to 0.

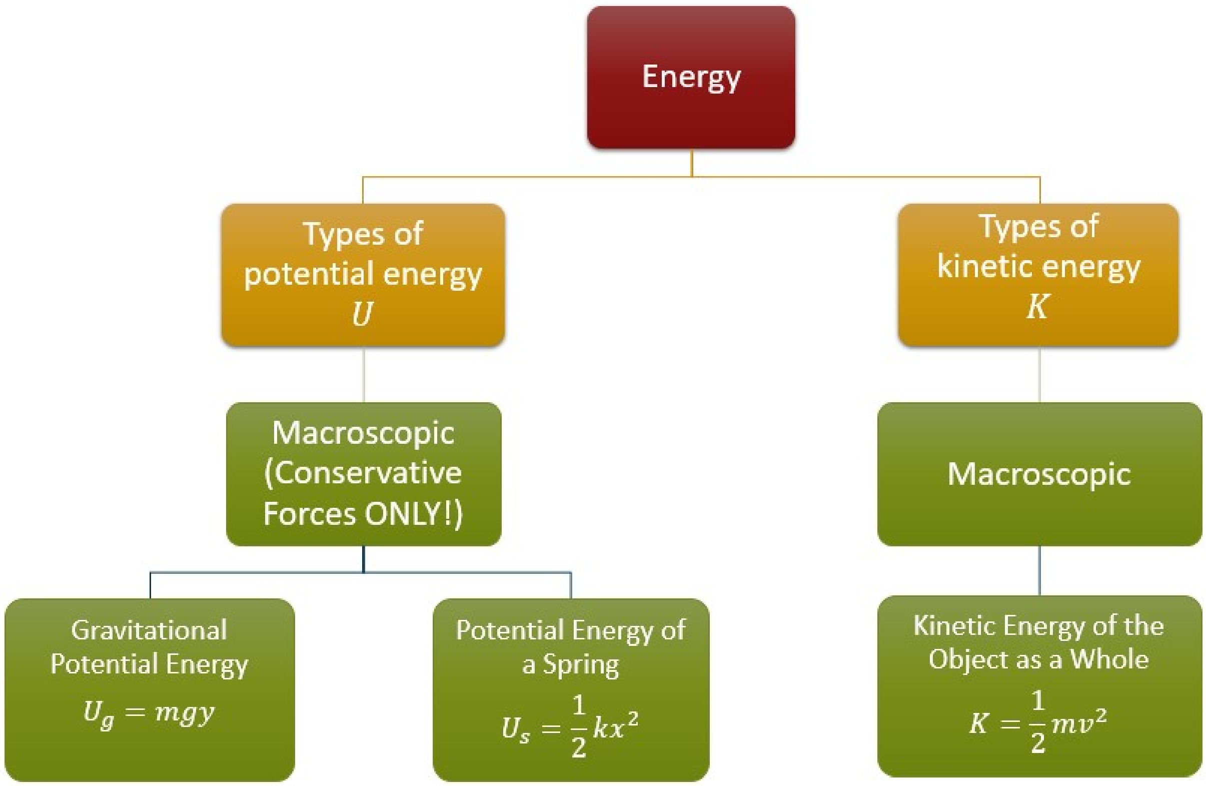
Thus, it seems that we have two different types of forces. We have forces for which the work done over a closed path is always equal to 0, an example of this is gravity, we call such forces conservative. We have another type of force for which the work done over a closed path is not equal to 0. The frictional force that we just saw is an example of this. Forces for which the work done over a closed path is not equal to 0 are called non-conservative forces. The definition of a conservative force is a force for which the work done by the force over a closed path is equal to 0. So, this is the statement you can use to determine if a force is conservative or non-conservative. Why do we care about this distinction between conservative and non-conservative forces? Because only conservative forces are associated with the potential energy.

To explore this idea, let's consider a block sliding down a frictionless ramp and a block just falling to the ground. Both blocks travel the same vertical distance h. In both cases, the work done by gravity is equal to the change in gravitational potential energy. The work done by gravity is written mathematically as a $-\Delta U$, or $-\Delta mgh$. The reason we can write the work done by gravity in terms of a potential energy is because gravity is a conservative force. Friction is not a conservative force; thus, we cannot write the work done by friction as the change in some type of frictional

potential energy. So, let’s summarize. We’ve seen that there are two classes of forces. Forces over which the work done over a closed path is zero, such as gravity, such forces are known as conservative, and for these types of forces we can describe the work done by the force as a change in the potential energy. $W = -\Delta U$. We also have forces for which the work done over a closed path is not equal to zero, such as friction. For these non-conservative forces, we cannot describe the work as in terms of a change in potential energy. There is no such thing as a potential energy for friction.

Organizing the Different Types of Macroscopic Energy (Mechanical Energy)

Now we have all of the different types of macroscopic energy that we will talk about in this course: kinetic energy $K = \frac{1}{2}mv^2$, gravitational potential energy $U_g = mgy$, and the potential energy of a spring $U_s = \frac{1}{2}kx^2$. These different types of energy can be organized as in the chart in the **figure**. Collectively these types of energy are called *mechanical energy.*



The different types of macroscopic (also called mechanical) energy

Review of Problem Solving with Conservation of Energy
Exercise:

UMASS AMHERST Instructor's Notes

Problem:

This section is also available as two videos on the UMass Physics 13X YouTube page. The links can be found here and here.

With what minimum speed must you toss a

$$140$$

gram ball straight up to hit the

$$14$$

meter high roof of a gymnasium if you release the ball

$$1.3$$

meters above the ground? With what speed does the ball hit the ground?

You can use conservation of energy to solve this problem.

What is the initial energy state of the ball? We have some kinetic energy and some potential energy, so we have both. How do we know we have kinetic energy? Because we throw the ball, if the ball has no initial Kinetic energy which means it's not moving that means it doesn't go up, it had to have had some kinetic energy for it to actually go up and had to have some initial velocity when we threw it. Does it have any initial potential energy? The ball starts

$$1.3$$

meters above the ground initially, this tells me it started out with some potential energy, it's already above the ground.

What is its final energy state in the perfect world in physics land? Does it have Kinetic energy at the roof? No, we're assuming it just touches the roof and has zero velocity at the roof for that moment in time, so its kinetic final energy is actually zero. All we have left is potential final energy.

Equation:

$$E_i = E_f$$

Equation:

$$K_i + U_i = K_f + U_f$$

Equation:

$$\frac{1}{2}mv^2 + mgh_i = 0 + mgh_f$$

Equation:

$$\frac{1}{2}v^2 + gh_i = gh_f$$

What speed does it hit the ground? Energy initial equals energy final, what's the initial energy state? My initial is the ball at the top of the ceiling. My final is just before it hits the ground. How fast does it hit the ground? It started from the roof, falls down. What is the energy state at the roof? It's all potential. What's the energy state the moment before it hits the ground? It's lost all its potential energy, and its converted into kinetic energy.

Equation:

$$E_i = E_f$$

Equation:

$$U_i = K_f$$

Equation:

$$mgh_i = \frac{1}{2}mv_f^2$$

Equation:

$$gh_i = \frac{1}{2}v_f^2$$

Equation:

$$v_f = \sqrt{2gh_i}$$

Equation:

$$v_f = \sqrt{2 \bullet 9.8 \bullet 14} = 16.6m/s$$

Your friends Frisbee has become stuck

$$26$$

meters above the ground in a tree. You want to dislodge the Frisbee by throwing a rock at it. The Frisbee is stuck pretty tight, so you figure the rock needs to be traveling at least

$$5.4m/s$$

when it hits the Frisbee. If you release the rock

$$1.6$$

meters above the ground, with what minimum speed must you throw it?

Energy initial has to equal energy final, what is my initial state of affairs? When I'm throwing the rock, that's my initial state of affairs. Do I have kinetic energy in the beginning? I must have it. How do I know I must have kinetic energy? Because I'm throwing the rock, so the rock has to have some initial velocity. Do I have any initial potential energy? Yes, because I started

$$1.6$$

meters above the ground. What's my final state of affairs? Do I have any kinetic energy at the end? When the rocks up there at the frisbee, does it have any kinetic energy? I know that it had to have a velocity,

$$5.4m/s$$

, I know that the moment before I hit the frisbee I had to have this velocity. Therefore, I know I had some kinetic energy up there. Do I have any final potential energy? Yes, because it is up in the tree.

$$E_i = E_f$$

Equation:

$$K_i + U_i = K_f + U_f$$

Equation:

$$\frac{1}{2}mv_i^2 + mgh_i = \frac{1}{2}mv_f^2 + mgh_f$$

Equation:

$$\frac{1}{2}v_i^2 + gh_i = \frac{1}{2}v_f^2 + gh_f$$

Equation:

$$v_i = \sqrt{2\bullet(\frac{1}{2}v_f^2 + gh_f - gh_i)}$$

What is a Wave
Exercise:

UMASS AMHERST Instructor's Notes

Problem:

This section is also available as a video on the UMass Physics 13X YouTube page. The link can be found here.

What about waves? Before we start talking about waves, it's probably best to give a few different examples of waves. If I ask you to think of a wave the first thing that probably would come to most your minds is a water wave, but we could also have waves on a string, or even sound waves. The most generic picture that a lot of you have, is probably some sort of sine or cosine shape traveling along, but this is not representative of all waves and we want our definition to be in terms of properties that apply to every possible wave that we can think of.

Let's go through a few questions and develop a definition of a wave.

Does the wave actually have to go anywhere, does a wave travel? No. Sure, most waves go somewhere, water waves travel across an ocean for example, but think of a guitar string, when you pluck it, certainly the string waves back and forth but the string doesn't go anywhere, the wave stands on the string. This is called a standing wave. So, traveling cannot be part of our definition of a wave.

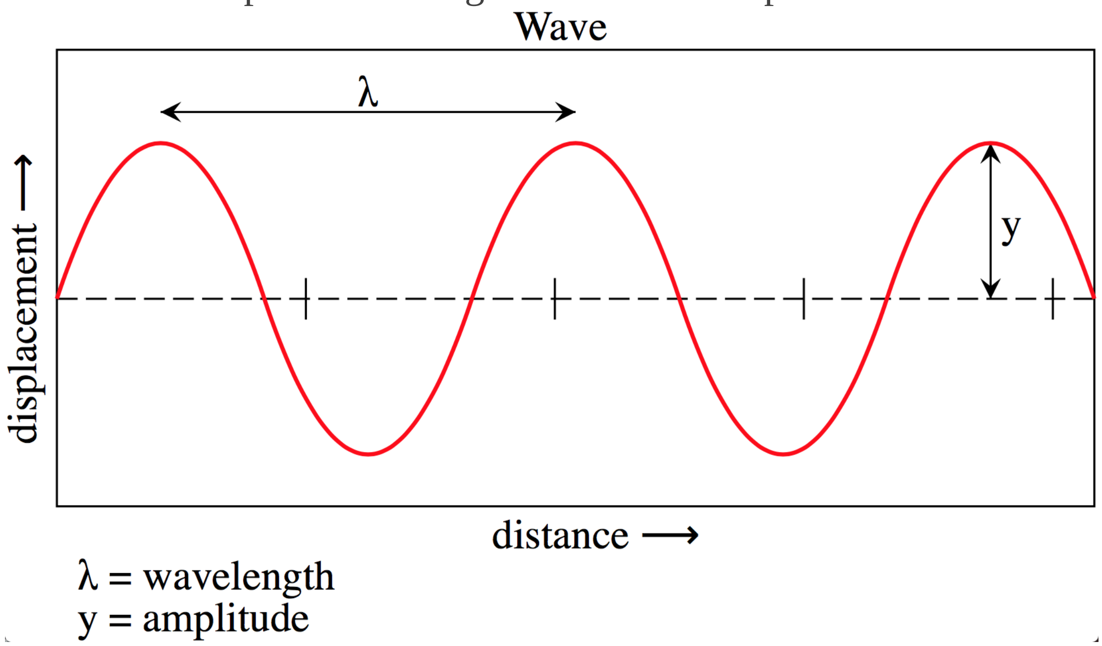
Does a wave have to be a repeating pattern? Again, not really. While this might be the image that a lot of you have in mind when I say the word wave, remember we can have just a single pulse going back and forth on a string.

Does the wave have to have up and down motion? Well again, no. This standard picture of a wave that you have in your head might but think of a sound wave? In a sound wave the molecules move back and forth in the

same direction as the waves motion, in contrast to the wave you probably have in mind where things move perpendicular to the waves motion.

Now we need a little bit of terminology. Waves that do wiggle perpendicular to the direction of motion of the wave are called transverse waves. These are the waves that you probably have in mind and these are the ones that were mostly going to be interested in.

The basic terminology of transverse waves, we'll introduce some more later, are that waves have a peak and a trough, and then the distance from the zero line to either a peak or a trough is called the amplitude.



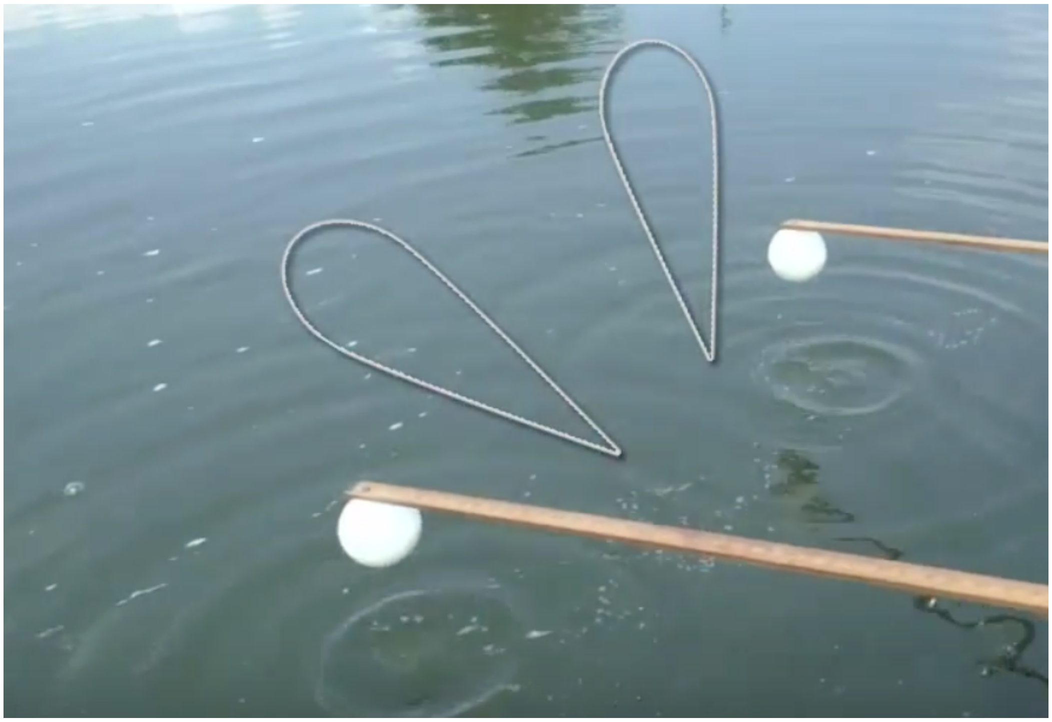
(Credit: Krishnavedala)

What other properties of a wave could we perhaps use? Can a wave bend around corners? We know that particles don't bend around corners, what about waves? Well, it turns out that waves do bend around corners. Think of a water wave, it spreads out, bending around the corner. In the video on the next slide we'll see some other important properties of waves that all waves do share.

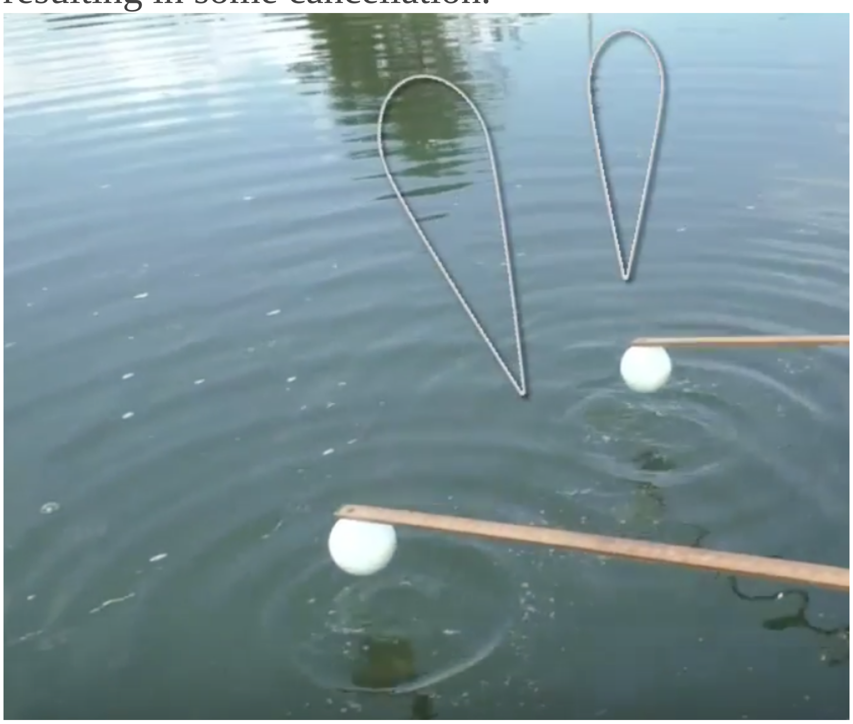
I go and just hit the water with a single stick, we see we get waves coming out in all directions, radiating away from the spot where the ball hits the water.

Things get a little bit more interesting, however, if we have two sources of waves going at the same time next to each other like so.

So now we get two waves, each radiating out from its source. In some places the waves line up peak to peak, or trough to trough, and add up, resulting in a larger wave at that point.



In other places, the peak of one wave meets the trough of the other, resulting in some cancellation.



This phenomenon of waves adding in some places and cancelling in others is known as interference and is a characteristic property of waves.

So, what is a wave? Well a wave is a disturbance that can, but doesn't necessarily have to, travel or it can just store energy and momentum. A traveling wave will carry energy from one position to another, think of the water wave that carries energy as it moves across the ocean and also momentum, as that wave hits you, you feel the momentum of the wave. For a standing wave, that energy is just being stored. When I pluck a guitar string, the energy is just being stored in the string and then ultimately releases this sound that we hear. A wave need not necessarily repeat, we can have simple pulse waves. But a wave can bend around corners, and waves of the same kind can interact with each other or with themselves, adding in some places and canceling in other places, through this idea of interference. These are the fundamental characteristics of waves. They don't exist at a particular place, they sort of spread out over a couple of different places and they can carry energy and momentum while bending around corners and interacting with themselves or other waves of the same kind.

Exercise:

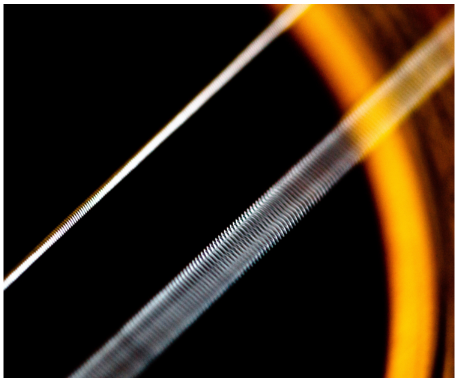
UMASS AMHERST Instructor's Notes

Problem:
To Summarize:

- Particles are localized in space, they don't bend around corners, but can carry energy and momentum.
- Waves on the other hand, are spread out in space, they are some kind of disturbance that can transfer or store energy and momentum.
- However, waves, unlike particles, can bend around corners and also waves can interact with themselves or other waves of the same kind through this phenomenon of interference.

Period and Frequency in Oscillations

- Observe the vibrations of a guitar string.
- Determine the frequency of oscillations.



The strings on this guitar vibrate at regular time intervals. (credit: JAR)

When you pluck a guitar string, the resulting sound has a steady tone and lasts a long time. Each successive vibration of the string takes the same time as the previous one. We define **periodic motion** to be a motion that repeats itself at regular time intervals, such as exhibited by the guitar string or by an object on a spring moving up and down. The time to complete one oscillation remains constant and is called the **period** T. Its units are usually seconds, but may be any convenient unit of time. The word period refers to the time for some event whether repetitive or not; but we shall be primarily interested in periodic motion, which is by definition repetitive. A concept closely related to period is the frequency of an event. For example, if you get a paycheck twice a month, the frequency of payment is two per month and the period between checks is half a month. **Frequency** f is defined to be the number of events per unit time. For periodic motion, frequency is the number of oscillations per unit time. The relationship between frequency and period is

Equation:

$$f = \frac{1}{T}.$$

The SI unit for frequency is the *cycle per second*, which is defined to be a *hertz* (Hz):

Equation:

$$1\text{ Hz} = 1\frac{\text{cycle}}{\text{sec}}\text{ or }1\text{ Hz} = \frac{1}{\text{s}}$$

A cycle is one complete oscillation. Note that a vibration can be a single or multiple event, whereas oscillations are usually repetitive for a significant number of cycles.

Example:
Determine the Frequency of Two Oscillations: Medical Ultrasound and the Period of Middle C

We can use the formulas presented in this module to determine both the frequency based on known oscillations and the oscillation based on a known frequency. Let's try one example of each. (a) A medical imaging device produces ultrasound by oscillating with a period of 0.400 µs. What is the frequency of this oscillation? (b) The frequency of middle C on a typical musical instrument is 264 Hz. What is the time for one complete oscillation?

Strategy

Both questions (a) and (b) can be answered using the relationship between period and frequency. In question (a), the period T is given and we are asked to find frequency f. In question (b), the frequency f is given and we are asked to find the period T.

Solution a

1. Substitute 0.400 µs for T in $f = \frac{1}{T}$:
   Equation:

$$f = \frac{1}{T} = \frac{1}{0.400 \times 10^{-6}\ \text{s}}.$$

Solve to find
Equation:

$$f = 2.50 \times 10^{6}\ \text{Hz}.$$

Discussion a
The frequency of sound found in (a) is much higher than the highest frequency that humans can hear and, therefore, is called ultrasound. Appropriate oscillations at this frequency generate ultrasound used for noninvasive medical diagnoses, such as observations of a fetus in the womb.
Solution b

1. Identify the known values:

   The time for one complete oscillation is the period T:
   Equation:

   $$f = \frac{1}{T}.$$

2. Solve for T:
   Equation:

   $$T = \frac{1}{f}.$$

3. Substitute the given value for the frequency into the resulting expression:
   Equation:

   $$T = \frac{1}{f} = \frac{1}{264\ \text{Hz}} = \frac{1}{264\ \text{cycles/s}} = 3.79 \times 10^{-3}\ \text{s} = 3.79\ \text{ms}.$$

Discussion
The period found in (b) is the time per cycle, but this value is often quoted as simply the time in convenient units (ms or milliseconds in this case).

Exercise:
Check your Understanding

Problem:

Identify an event in your life (such as receiving a paycheck) that occurs regularly. Identify both the period and frequency of this event.

Solution:

I visit my parents for dinner every other Sunday. The frequency of my visits is 26 per calendar year. The period is two weeks.

Section Summary

- Periodic motion is a repetitious oscillation.
- The time for one oscillation is the period T.
- The number of oscillations per unit time is the frequency f.
- These quantities are related by
  Equation:

$$f = \frac{1}{T}.$$

Glossary

period
: time it takes to complete one oscillation

periodic motion

motion that repeats itself at regular time intervals

frequency

number of events per unit of time

Waves

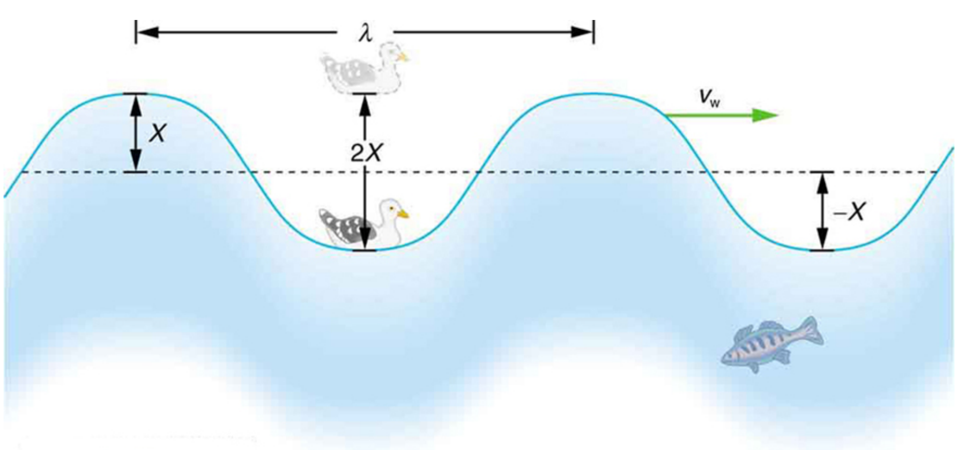
- State the characteristics of a wave.
- Calculate the velocity of wave propagation.

Waves in the ocean behave similarly to all other types of waves. (credit: Steve Jurveston, Flickr)

What do we mean when we say something is a wave? The most intuitive and easiest wave to imagine is the familiar water wave. More precisely, a **wave** is a disturbance that propagates, or moves from the place it was created. For water waves, the disturbance is in the surface of the water, perhaps created by a rock thrown into a pond or by a swimmer splashing the surface repeatedly. For sound waves, the disturbance is a change in air pressure, perhaps created by the oscillating cone inside a speaker. For earthquakes, there are several types of disturbances, including disturbance of Earth’s surface and pressure disturbances under the surface. Even radio waves are most easily understood using an analogy with water waves. Visualizing water waves is useful because there is more to it than just a mental image. Water waves exhibit characteristics common to all waves, such as amplitude, period, frequency and energy. All wave characteristics can be described by a small set of underlying principles.

A wave is a disturbance that propagates, or moves from the place it was created. The simplest waves repeat themselves for several cycles and are associated with simple harmonic motion. Let us start by considering the simplified water wave in [link]. The wave is an up and down disturbance of the water surface. It causes a sea gull to move up and down in simple harmonic motion as the wave crests and troughs (peaks and valleys) pass under the bird. The time for one complete up and down motion is the wave's period T. The wave's frequency is $f = 1/T$, as usual. The wave itself moves to the right in the figure. This movement of the wave is actually the disturbance moving to the right, not the water itself (or the bird would move to the right). We define **wave velocity** v_{w} to be the speed at which the disturbance moves. Wave velocity is sometimes also called the *propagation velocity or propagation speed*, because the disturbance propagates from one location to another.

Note:
Misconception Alert
Many people think that water waves push water from one direction to another. In fact, the particles of water tend to stay in one location, save for moving up and down due to the energy in the wave. The energy moves forward through the water, but the water stays in one place. If you feel yourself pushed in an ocean, what you feel is the energy of the wave, not a rush of water.



An idealized ocean wave passes under a sea gull that bobs up and down in simple harmonic motion. The wave has a wavelength λ, which is the distance between adjacent identical parts of the wave. The up and down disturbance of the surface propagates parallel to the surface at a speed v_w.

The water wave in the figure also has a length associated with it, called its **wavelength** λ, the distance between adjacent identical parts of a wave. (λ is the distance parallel to the direction of propagation.) The speed of propagation v_w is the distance the wave travels in a given time, which is one wavelength in the time of one period. In equation form, that is

Equation:

$$v_w = \frac{\lambda}{T}$$

or

Equation:

$$v_w = f\lambda.$$

This fundamental relationship holds for all types of waves. For water waves, v_w is the speed of a surface wave; for sound, v_w is the speed of sound; and for visible light, v_w is the speed of light, for example.

Note:

Take-Home Experiment: Waves in a Bowl

Fill a large bowl or basin with water and wait for the water to settle so there are no ripples. Gently drop a cork into the middle of the bowl. Estimate the wavelength and period of oscillation of the water wave that propagates away from the cork. Remove the cork from the bowl and wait

for the water to settle again. Gently drop the cork at a height that is different from the first drop. Does the wavelength depend upon how high above the water the cork is dropped?

Example:
Calculate the Velocity of Wave Propagation: Gull in the Ocean
Calculate the wave velocity of the ocean wave in [link] if the distance between wave crests is 10.0 m and the time for a sea gull to bob up and down is 5.00 s.
Strategy
We are asked to find v_{w}. The given information tells us that $\lambda = 10.0\text{ m}$ and $T = 5.00\text{ s}$. Therefore, we can use $v_{\text{w}} = \frac{\lambda}{T}$ to find the wave velocity.
Solution

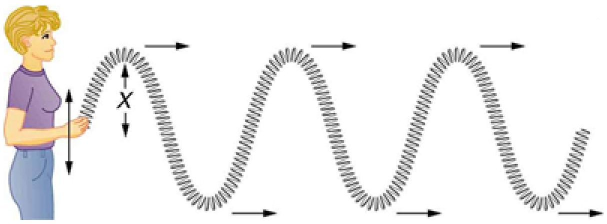
1. Enter the known values into $v_{\text{w}} = \frac{\lambda}{T}$:
   Equation:
   $$v_{\text{w}} = \frac{10.0\text{ m}}{5.00\text{ s}}.$$
2. Solve for v_{w} to find $v_{\text{w}} = 2.00\text{ m/s}$.

Discussion
This slow speed seems reasonable for an ocean wave. Note that the wave moves to the right in the figure at this speed, not the varying speed at which the sea gull moves up and down.

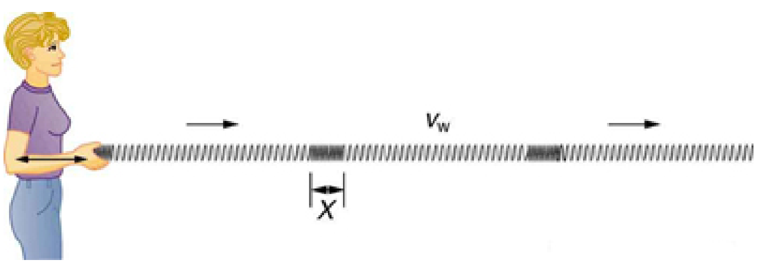
Transverse and Longitudinal Waves

A simple wave consists of a periodic disturbance that propagates from one place to another. The wave in [link] propagates in the horizontal direction while the surface is disturbed in the vertical direction. Such a wave is called a **transverse wave** or shear wave; in such a wave, the disturbance is perpendicular to the direction of propagation. In contrast, in a **longitudinal**

wave or compressional wave, the disturbance is parallel to the direction of propagation. [link] shows an example of a longitudinal wave. The size of the disturbance is its amplitude X and is completely independent of the speed of propagation v_w.



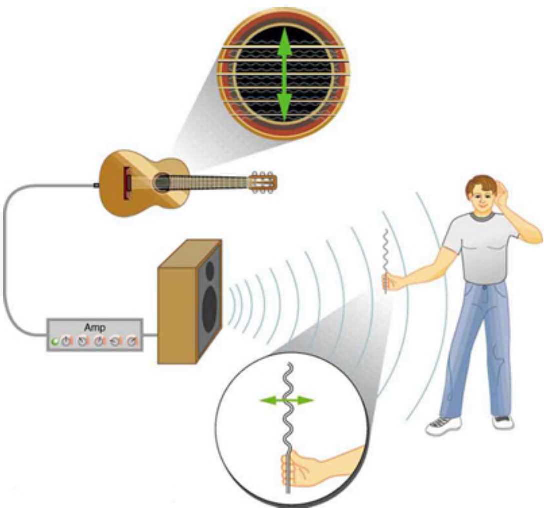
In this example of a transverse wave, the wave propagates horizontally, and the disturbance in the cord is in the vertical direction.



In this example of a longitudinal wave, the wave propagates horizontally, and the disturbance in the cord is also in the horizontal direction.

Waves may be transverse, longitudinal, or *a combination of the two*. (Water waves are actually a combination of transverse and longitudinal. The simplified water wave illustrated in [link] shows no longitudinal motion of the bird.) The waves on the strings of musical instruments are transverse—so are electromagnetic waves, such as visible light.

Sound waves in air and water are longitudinal. Their disturbances are periodic variations in pressure that are transmitted in fluids. Fluids do not have appreciable shear strength, and thus the sound waves in them must be longitudinal or compressional. Sound in solids can be both longitudinal and transverse.



The wave on a guitar string is transverse. The sound wave rattles a sheet of paper in a direction that shows the sound wave is longitudinal.

Earthquake waves under Earth's surface also have both longitudinal and transverse components (called compressional or P-waves and shear or S-waves, respectively). These components have important individual characteristics—they propagate at different speeds, for example.

Earthquakes also have surface waves that are similar to surface waves on water.

Exercise:

Check Your Understanding

Problem:

Why is it important to differentiate between longitudinal and transverse waves?

Solution:

In the different types of waves, energy can propagate in a different direction relative to the motion of the wave. This is important to understand how different types of waves affect the materials around them.

Note:

PhET Explorations: Wave on a String

Watch a string vibrate in slow motion. Wiggle the end of the string and make waves, or adjust the frequency and amplitude of an oscillator. Adjust the damping and tension. The end can be fixed, loose, or open.
https://phet.colorado.edu/sims/html/wave-on-a-string/latest/wave-on-a-string_en.html

Section Summary

- A wave is a disturbance that moves from the point of creation with a wave velocity v_{w}.
- A wave has a wavelength λ, which is the distance between adjacent identical parts of the wave.
- Wave velocity and wavelength are related to the wave's frequency and period by $v_{\text{w}} = \frac{\lambda}{T}$ or $v_{\text{w}} = f\lambda$.

- A transverse wave has a disturbance perpendicular to its direction of propagation, whereas a longitudinal wave has a disturbance parallel to its direction of propagation.

Glossary

longitudinal wave
: a wave in which the disturbance is parallel to the direction of propagation

transverse wave
: a wave in which the disturbance is perpendicular to the direction of propagation

wave velocity
: the speed at which the disturbance moves. Also called the propagation velocity or propagation speed

wavelength
: the distance between adjacent identical parts of a wave

Detailed Description of a Wave
Exercise:

UMASS AMHERST Instructor's Notes

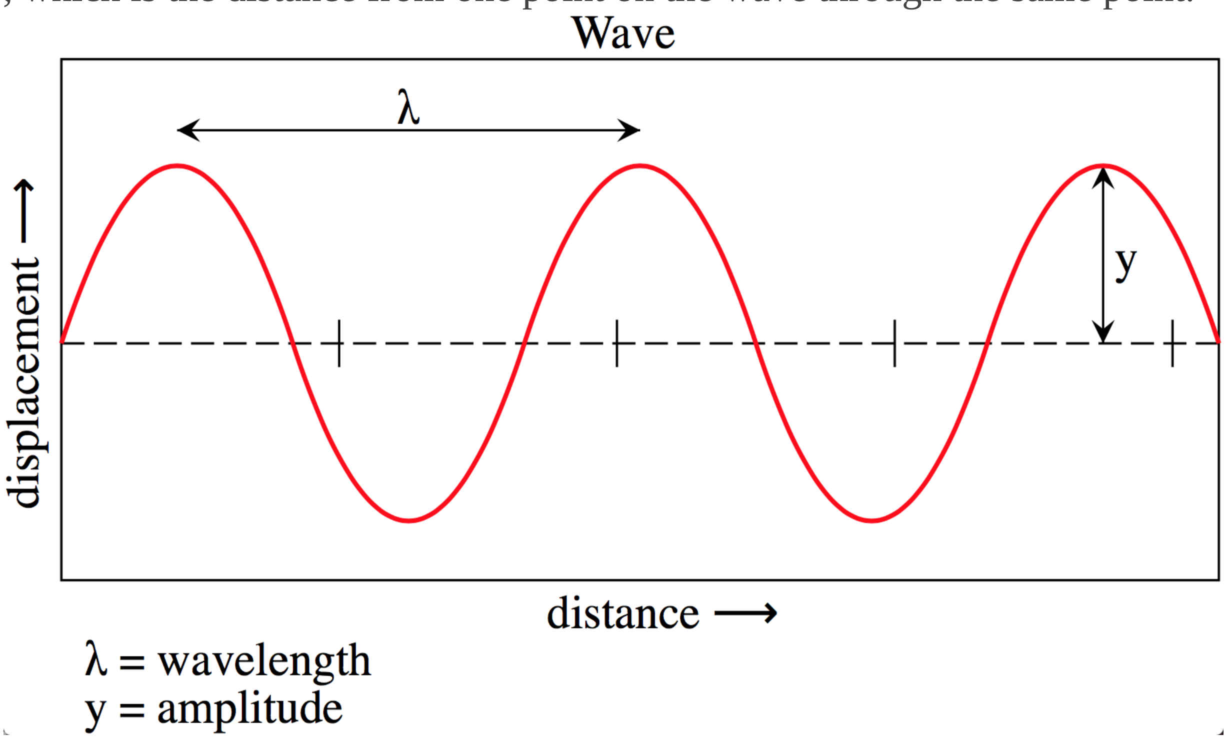
Problem:

This section is also available as a video on the UMass Physics 13X YouTube page. The link can be found here.

Let’s begin by thinking about the one thing we have already figured out, which is that light has some wave-like properties. In the figure below we have a wave, and we can talk about the wavelength,

$$\lambda$$

, which is the distance from one point on the wave through the same point.



Wave Diagram (Credit: Krishnavedala)

Wavelength could be from peak to peak or from zero to zero or from trough to trough. All those distances are the same, and the wavelength is measured in meters.

We also have the idea of the amplitude. The difference from either peak to the middle average or from trough to the middle average is what we call the amplitude.

In addition to wavelength and amplitude, we can also talk about how long it takes for a point to go up and down. Think about one point on the wave, bouncing up and down. We can talk about how long it takes for that point on the wave to go from trough to peak to trough, the amount of time that takes is called the period,

$$T$$

, and will be measured in seconds.

IN For this class, we will work in SI, International System of Units, we will not work in, what I call barbaric units, there shall be no inches. Meters, kilograms, and seconds will be the norm. On the Moodle page there's a document of math I expect you to know, it's things like trigonometry and area of a circle, but I also expect you to know the SI prefixes Nano - Giga. I will not give you these, and on an exam if you come to the TA's and ask how big a micrometer is, we're going to have to say tough cookie, that's something you were supposed to know.

The period is the time it takes for a point on the wave to go up and down, measured in seconds, but I can also count how many that point oscillates in one second. I could say how many oscillations per second this point on this wave make? That quantity is known as the frequency and its value is
Equation:

$$f = \frac{1}{T}$$

, the unit of frequency is how many per second or one over seconds, which is called Hertz.

We will use these basic terms for all of the waves we discuss, electrons and light.

We know that wavelength is measured in meters and we know that frequency is in Hertz, or 1 over seconds, so

$$\lambda \bullet f$$

will be meters over seconds which is a velocity. What's the only velocity that we could have? The speed of the wave, it's the only speed we could talk about, so we have the speed of the wave is going to be
Equation:

$$v = \lambda f$$

If we know that the speed of a wave is fixed, for example, light travels at a fixed speed, then if the frequency goes up, what's the wavelength going to do?

We know that

$$v = \lambda f$$

. The speed is fixed, if the frequency goes up, and lambda times f has to be the same thing, then that is going to tell me that the wavelength must go down.

IN This question is based on mathematical reasoning using symbols and not numbers. Remember one of the goals for this course was working in symbols and not numbers, so here's an example of that.
Exercise:

UMASS AMHERST Instructor's Notes

Problem:

This question is based on mathematical reasoning using symbols and not numbers. Remember one of the goals for this course was working in symbols and not numbers, so here's an example of that.

Power

- Calculate power by calculating changes in energy over time.
- Examine power consumption and calculations of the cost of energy consumed.

What is Power?

Power—the word conjures up many images: a professional football player muscling aside his opponent, a dragster roaring away from the starting line, a volcano blowing its lava into the atmosphere, or a rocket blasting off, as in [link].

This powerful rocket on the Space Shuttle *Endeavor* did work and consumed energy at a very high rate. (credit: NASA)

These images of power have in common the rapid performance of work, consistent with the scientific definition of **power** (P) as the rate at which work is done.

Note:
Power
Power is the rate at which work is done.
Equation:

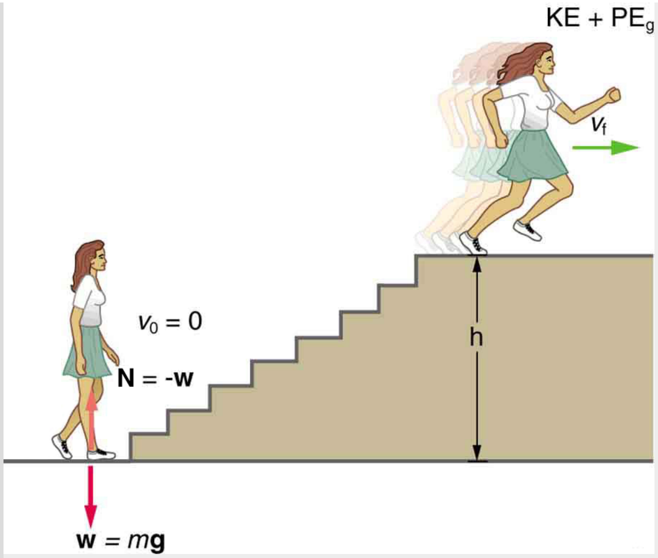
$$P = \frac{W}{t}$$

The SI unit for power is the **watt** (W), where 1 watt equals 1 joule/second $(1\text{ W} = 1\text{ J/s})$.

Because work is energy transfer, power is also the rate at which energy is expended. A 60-W light bulb, for example, expends 60 J of energy per second. Great power means a large amount of work or energy developed in a short time. For example, when a powerful car accelerates rapidly, it does a large amount of work and consumes a large amount of fuel in a short time.

Calculating Power from Energy

Example:
Calculating the Power to Climb Stairs
What is the power output for a 60.0-kg woman who runs up a 3.00 m high flight of stairs in 3.50 s, starting from rest but having a final speed of 2.00 m/s? (See [link].)



When this woman runs upstairs starting from rest, she converts the chemical energy originally from food into kinetic energy and gravitational potential energy. Her power output depends on how fast she does this.

Strategy and Concept
The work going into mechanical energy is $W = \mathrm{KE} + \mathrm{PE}$. At the bottom of the stairs, we take both KE and $\mathrm{PE_g}$ as initially zero; thus, $W = \mathrm{KE_f} + \mathrm{PE_g} = \frac{1}{2}m{v_\mathrm{f}}^2 + mgh$, where h is the vertical height of the stairs. Because all terms are given, we can calculate W and then divide it by time to get power.
Solution
Substituting the expression for W into the definition of power given in the previous equation, $P = W/t$ yields
Equation:

$$P = \frac{W}{t} = \frac{\frac{1}{2}m{v_\mathrm{f}}^2 + mgh}{t}.$$

Entering known values yields

Equation:

$$\begin{aligned} P &= \frac{0.5(60.0\text{ kg})(2.00\text{ m/s})^2+(60.0\text{ kg})\left(9.80\text{ m/s}^2\right)(3.00\text{ m})}{3.50\text{ s}} \\ &= \frac{120\text{ J}+1764\text{ J}}{3.50\text{ s}} \\ &= 538\text{ W}. \end{aligned}$$

Discussion

The woman does 1764 J of work to move up the stairs compared with only 120 J to increase her kinetic energy; thus, most of her power output is required for climbing rather than accelerating.

It is impressive that this woman's useful power output is slightly less than 1 **horsepower** $(1\text{ hp} = 746\text{ W})$! People can generate more than a horsepower with their leg muscles for short periods of time by rapidly converting available blood sugar and oxygen into work output. (A horse can put out 1 hp for hours on end.) Once oxygen is depleted, power output decreases and the person begins to breathe rapidly to obtain oxygen to metabolize more food—this is known as the *aerobic* stage of exercise. If the woman climbed the stairs slowly, then her power output would be much less, although the amount of work done would be the same.

Note:

Making Connections: Take-Home Investigation—Measure Your Power Rating

Determine your own power rating by measuring the time it takes you to climb a flight of stairs. We will ignore the gain in kinetic energy, as the above example showed that it was a small portion of the energy gain. Don't expect that your output will be more than about 0.5 hp.

Examples of Power

Examples of power are limited only by the imagination, because there are as many types as there are forms of work and energy. (See [link] for some examples.) Sunlight reaching Earth's surface carries a maximum power of about 1.3 kilowatts per square meter $(\mathrm{kW/m^2})$. A tiny fraction of this is retained by Earth over the long term. Our consumption rate of fossil fuels is far greater than the rate at which they are stored, so it is inevitable that they will be depleted. Power implies that energy is transferred, perhaps changing form. It is never possible to change one form completely into another without losing some of it as thermal energy. For example, a 60-W incandescent bulb converts only 5 W of electrical power to light, with 55 W dissipating into thermal energy. Furthermore, the typical electric power plant converts only 35 to 40% of its fuel into electricity. The remainder becomes a huge amount of thermal energy that must be dispersed as heat transfer, as rapidly as it is created. A coal-fired power plant may produce 1000 megawatts; 1 megawatt (MW) is 10^6 W of electric power. But the power plant consumes chemical energy at a rate of about 2500 MW, creating heat transfer to the surroundings at a rate of 1500 MW. (See [link].)

Tremendous amounts of electric power are generated by coal-fired power plants such as this one in China, but an even larger amount of power goes into heat transfer to the surroundings.

The large cooling towers here are needed to transfer heat as rapidly as it is produced. The transfer of heat is not unique to coal plants but is an unavoidable consequence of generating electric power from any fuel—nuclear, coal, oil, natural gas, or the like. (credit: Kleinolive, Wikimedia Commons)

Object or Phenomenon	**Power in Watts**
Supernova (at peak)	5×10^{37}
Milky Way galaxy	10^{37}
Crab Nebula pulsar	10^{28}
The Sun	4×10^{26}

Object or Phenomenon	Power in Watts
Volcanic eruption (maximum)	4×10^{15}
Lightning bolt	2×10^{12}
Nuclear power plant (total electric and heat transfer)	3×10^{9}
Aircraft carrier (total useful and heat transfer)	10^{8}
Dragster (total useful and heat transfer)	2×10^{6}
Car (total useful and heat transfer)	8×10^{4}
Football player (total useful and heat transfer)	5×10^{3}
Clothes dryer	4×10^{3}
Person at rest (all heat transfer)	100

Object or Phenomenon	Power in Watts
Typical incandescent light bulb (total useful and heat transfer)	60
Heart, person at rest (total useful and heat transfer)	8
Electric clock	3
Pocket calculator	10^{-3}

Power Output or Consumption

Power and Energy Consumption

We usually have to pay for the energy we use. It is interesting and easy to estimate the cost of energy for an electrical appliance if its power consumption rate and time used are known. The higher the power consumption rate and the longer the appliance is used, the greater the cost of that appliance. The power consumption rate is $P = W/t = E/t$, where E is the energy supplied by the electricity company. So the energy consumed over a time t is
Equation:

$$E = \text{Pt}.$$

Electricity bills state the energy used in units of **kilowatt-hours** $(\text{kW} \cdot \text{h})$, which is the product of power in kilowatts and time in hours. This unit is convenient because electrical power consumption at the kilowatt level for hours at a time is typical.

Example:
Calculating Energy Costs
What is the cost of running a 0.200-kW computer 6.00 h per day for 30.0 d if the cost of electricity is $0.120 per $\mathrm{kW \cdot h}$?
Strategy
Cost is based on energy consumed; thus, we must find E from $E = \mathrm{Pt}$ and then calculate the cost. Because electrical energy is expressed in $\mathrm{kW \cdot h}$, at the start of a problem such as this it is convenient to convert the units into kW and hours.
Solution
The energy consumed in $\mathrm{kW \cdot h}$ is
Equation:

$$\begin{aligned} E &= \mathrm{Pt} = (0.200\,\mathrm{kW})(6.00\,\mathrm{h/d})(30.0\,\mathrm{d}) \\ &= 36.0\,\mathrm{kW \cdot h}, \end{aligned}$$

and the cost is simply given by
Equation:

$$\text{cost} = (36.0\ \mathrm{kW \cdot h})(\$0.120\text{ per }\mathrm{kW \cdot h}) = \$4.32\text{ per month.}$$

Discussion
The cost of using the computer in this example is neither exorbitant nor negligible. It is clear that the cost is a combination of power and time. When both are high, such as for an air conditioner in the summer, the cost is high.

The motivation to save energy has become more compelling with its ever-increasing price. Armed with the knowledge that energy consumed is the product of power and time, you can estimate costs for yourself and make the necessary value judgments about where to save energy. Either power or time must be reduced. It is most cost-effective to limit the use of high-power devices that normally operate for long periods of time, such as water heaters and air conditioners. This would not include relatively high power devices like toasters, because they are on only a few minutes per day. It would also not include electric clocks, in spite of their 24-hour-per-day

usage, because they are very low power devices. It is sometimes possible to use devices that have greater efficiencies—that is, devices that consume less power to accomplish the same task. One example is the compact fluorescent light bulb, which produces over four times more light per watt of power consumed than its incandescent cousin.

Modern civilization depends on energy, but current levels of energy consumption and production are not sustainable. The likelihood of a link between global warming and fossil fuel use (with its concomitant production of carbon dioxide), has made reduction in energy use as well as a shift to non-fossil fuels of the utmost importance. Even though energy in an isolated system is a conserved quantity, the final result of most energy transformations is waste heat transfer to the environment, which is no longer useful for doing work. As we will discuss in more detail in Thermodynamics, the potential for energy to produce useful work has been "degraded" in the energy transformation.

Section Summary

- Power is the rate at which work is done, or in equation form, for the average power P for work W done over a time t, $P = W/t$.
- The SI unit for power is the watt (W), where $1\text{ W} = 1\text{ J/s}$.
- The power of many devices such as electric motors is also often expressed in horsepower (hp), where $1\text{ hp} = 746\text{ W}$.

Glossary

power
: the rate at which work is done

watt
: (W) SI unit of power, with $1\text{ W} = 1\text{ J/s}$

horsepower
: an older non-SI unit of power, with $1\text{ hp} = 746\text{ W}$

kilowatt-hour

$(\mathrm{kW} \cdot \mathrm{h})$ unit used primarily for electrical energy provided by electric utility companies

Energy in Waves: Intensity

- Calculate the intensity and the power of rays and waves.

The destructive effect of an earthquake is palpable evidence of the energy carried in these waves. The Richter scale rating of earthquakes is related to both their amplitude and the energy they carry. (credit: Petty Officer 2nd Class Candice Villarreal, U.S. Navy)

All waves carry energy. The energy of some waves can be directly observed. Earthquakes can shake whole cities to the ground, performing the work of thousands of wrecking balls.

Loud sounds pulverize nerve cells in the inner ear, causing permanent hearing loss. Ultrasound is used for deep-heat treatment of muscle strains. A laser beam can burn away a malignancy. Water waves chew up beaches.

The amount of energy in a wave is related to its amplitude. Large-amplitude earthquakes produce large ground displacements. Loud sounds have higher pressure amplitudes and come from larger-amplitude source vibrations than

soft sounds. Large ocean breakers churn up the shore more than small ones. More quantitatively, a wave is a displacement that is resisted by a restoring force. The larger the displacement x, the larger the force $F = \mathrm{kx}$ needed to create it. Because work W is related to force multiplied by distance (Fx) and energy is put into the wave by the work done to create it, the energy in a wave is related to amplitude. In fact, a wave's energy is directly proportional to its amplitude squared because

Equation:

$$W \propto \mathrm{Fx} = \mathrm{kx}^2.$$

Exercise:

UMASS AMHERST Instructor's Notes

Problem:

The fact that energy is proportional to amplitude squared is important and we will come back to it later!

The energy effects of a wave depend on time as well as amplitude. For example, the longer deep-heat ultrasound is applied, the more energy it transfers. Waves can also be concentrated or spread out. Sunlight, for example, can be focused to burn wood. Earthquakes spread out, so they do less damage the farther they get from the source. In both cases, changing the area the waves cover has important effects. All these pertinent factors are included in the definition of **intensity** I as power per unit area:

Equation:

$$I = \frac{P}{A}$$

where P is the power carried by the wave through area A. The definition of intensity is valid for any energy in transit, including that carried by waves. The SI unit for intensity is watts per square meter ($\mathrm{W/m^2}$). For example,

infrared and visible energy from the Sun impinge on Earth at an intensity of $1300 \mathrm{~W/m^2}$ just above the atmosphere. There are other intensity-related units in use, too. The most common is the decibel. For example, a 90 decibel sound level corresponds to an intensity of $10^{-3} \mathrm{~W/m^2}$. (This quantity is not much power per unit area considering that 90 decibels is a relatively high sound level. Decibels will be discussed in some detail in a later chapter.

Example:
Calculating intensity and power: How much energy is in a ray of sunlight?
The average intensity of sunlight on Earth's surface is about $700 \mathrm{~W/m^2}$.
(a) Calculate the amount of energy that falls on a solar collector having an area of $0.500 \mathrm{~m^2}$ in 4.00 h.
(b) What intensity would such sunlight have if concentrated by a magnifying glass onto an area 200 times smaller than its own?
Strategy a
Because power is energy per unit time or $P = \frac{E}{t}$, the definition of intensity can be written as $I = \frac{P}{A} = \frac{E/t}{A}$, and this equation can be solved for E with the given information.
Solution a

1. Begin with the equation that states the definition of intensity:
   Equation:

$$I = \frac{P}{A}.$$

2. Replace P with its equivalent E/t:
   Equation:

$$I = \frac{E/t}{A}.$$

3. Solve for E:

Equation:

$$E = \mathrm{IAt}.$$

4. Substitute known values into the equation:
   Equation:

$$E = \left(700\ \mathrm{W/m^2}\right)\left(0.500\ \mathrm{m^2}\right)\left[(4.00\ \mathrm{h})(3600\ \mathrm{s/h})\right].$$

5. Calculate to find E and convert units:
   Equation:

$$5.04 \times 10^6\ \mathrm{J},$$

Discussion a
The energy falling on the solar collector in 4 h in part is enough to be useful—for example, for heating a significant amount of water.
Strategy b
Taking a ratio of new intensity to old intensity and using primes for the new quantities, we will find that it depends on the ratio of the areas. All other quantities will cancel.
Solution b

1. Take the ratio of intensities, which yields:
   Equation:

$$\frac{I\prime}{I} = \frac{P\prime/A\prime}{P/A} = \frac{A}{A\prime}\left(\text{The powers cancel because } P\prime = P\right).$$

2. Identify the knowns:
   Equation:

$$A = 200A\prime,$$

Equation:

$$\frac{I\prime}{I} = 200.$$

3. Substitute known quantities:
   Equation:

$$I\prime = 200I = 200\left(700\ \text{W/m}^2\right).$$

4. Calculate to find $I\prime$:
   Equation:

$$I\prime = 1.40 \times 10^5\ \text{W/m}^2.$$

Discussion b
Decreasing the area increases the intensity considerably. The intensity of the concentrated sunlight could even start a fire.

Example:
Determine the combined intensity of two waves: Perfect constructive interference
If two identical waves, each having an intensity of $1.00\ \text{W/m}^2$, interfere perfectly constructively, what is the intensity of the resulting wave?
Strategy
We know from Superposition and Interference that when two identical waves, which have equal amplitudes X, interfere perfectly constructively, the resulting wave has an amplitude of $2X$. Because a wave's intensity is proportional to amplitude squared, the intensity of the resulting wave is four times as great as in the individual waves.
Solution

1. Recall that intensity is proportional to amplitude squared.
2. Calculate the new amplitude:
   Equation:

$$I\prime \propto (X\prime)^2 = (2X)^2 = 4X^2.$$

3. Recall that the intensity of the old amplitude was:
   Equation:

$$I \propto X^2.$$

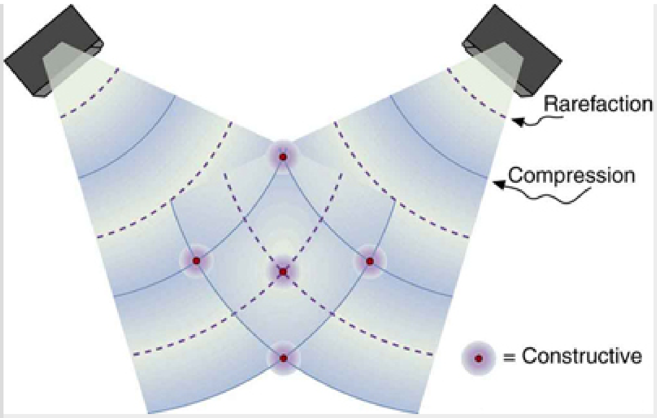
4. Take the ratio of new intensity to the old intensity. This gives:
   Equation:

$$\frac{I\prime}{I} = 4.$$

5. Calculate to find $I\prime$:
   Equation:

$$I\prime = 4I = 4.00\ \mathrm{W/m^2}.$$

Discussion
The intensity goes up by a factor of 4 when the amplitude doubles. This answer is a little disquieting. The two individual waves each have intensities of $1.00\ \mathrm{W/m^2}$, yet their sum has an intensity of $4.00\ \mathrm{W/m^2}$, which may appear to violate conservation of energy. This violation, of course, cannot happen. What does happen is intriguing. The area over which the intensity is $4.00\ \mathrm{W/m^2}$ is much less than the area covered by the two waves before they interfered. There are other areas where the intensity is zero. The addition of waves is not as simple as our first look in Superposition and Interference suggested. We actually get a pattern of both constructive interference and destructive interference whenever two waves are added. For example, if we have two stereo speakers putting out $1.00\ \mathrm{W/m^2}$ each, there will be places in the room where the intensity is $4.00\ \mathrm{W/m^2}$, other places where the intensity is zero, and others in between. [link] shows what this interference might look like. We will pursue interference patterns elsewhere in this text.



These stereo speakers produce both constructive interference and destructive interference in the room, a property common to the superposition of all types of waves. The shading is proportional to intensity.

Exercise:
Check Your Understanding

Problem:

Which measurement of a wave is most important when determining the wave's intensity?

Solution:

Amplitude, because a wave’s energy is directly proportional to its amplitude squared.

Section Summary

Intensity is defined to be the power per unit area:

$I = \frac{P}{A}$ and has units of W/m^2.

Conceptual Questions

Exercise:

Problem:

Give one example of a transverse wave and another of a longitudinal wave, being careful to note the relative directions of the disturbance and wave propagation in each.

Exercise:

Problem:

What is the difference between propagation speed and the frequency of a wave? Does one or both affect wavelength? If so, how?

Exercise:

Problem:

Most electrical appliances are rated in watts. Does this rating depend on how long the appliance is on? (When off, it is a zero-watt device.) Explain in terms of the definition of power.

Exercise:

Problem:

Explain, in terms of the definition of power, why energy consumption is sometimes listed in kilowatt-hours rather than joules. What is the relationship between these two energy units?

Exercise:

Problem:

A spark of static electricity, such as that you might receive from a doorknob on a cold dry day, may carry a few hundred watts of power. Explain why you are not injured by such a spark.

Exercise:

Problem:

Two identical waves undergo pure constructive interference. Is the resultant intensity twice that of the individual waves? Explain your answer.

Exercise:

Problem:

Circular water waves decrease in amplitude as they move away from where a rock is dropped. Explain why.

Problems & Exercises

Exercise:

Problem: What is the period of 60.0 Hz electrical power?

Solution:

16.7 ms

Exercise:

Problem:

If your heart rate is 150 beats per minute during strenuous exercise, what is the time per beat in units of seconds?

Solution:

0.400 s/beats

Exercise:

Problem:

Find the frequency of a tuning fork that takes 2.50×10^{-3} s to complete one oscillation.

Solution:

400 Hz

Exercise:

Problem:

A stroboscope is set to flash every 8.00×10^{-5} s. What is the frequency of the flashes?

Solution:

12,500 Hz

Exercise:

Problem:

A tire has a tread pattern with a crevice every 2.00 cm. Each crevice makes a single vibration as the tire moves. What is the frequency of these vibrations if the car moves at 30.0 m/s?

Solution:

1.50 kHz

Exercise:

Problem:

Storms in the South Pacific can create waves that travel all the way to the California coast, which are 12,000 km away. How long does it take them if they travel at 15.0 m/s?

Solution:
Equation:

$$t = 9.26\ \text{d}$$

Exercise:

Problem:

Waves on a swimming pool propagate at 0.750 m/s. You splash the water at one end of the pool and observe the wave go to the opposite end, reflect, and return in 30.0 s. How far away is the other end of the pool?

Exercise:

Problem:

Wind gusts create ripples on the ocean that have a wavelength of 5.00 cm and propagate at 2.00 m/s. What is their frequency?

Solution:
Equation:

$$f = 40.0\ \text{Hz}$$

Exercise:

Problem:

How many times a minute does a boat bob up and down on ocean waves that have a wavelength of 40.0 m and a propagation speed of 5.00 m/s?

Exercise:

Problem:

Scouts at a camp shake the rope bridge they have just crossed and observe the wave crests to be 8.00 m apart. If they shake it the bridge twice per second, what is the propagation speed of the waves?

Solution:
Equation:

$$v_{\mathrm{w}} = 16.0 \text{ m/s}$$

Exercise:

Problem:

What is the wavelength of the waves you create in a swimming pool if you splash your hand at a rate of 2.00 Hz and the waves propagate at 0.800 m/s?

Exercise:

Problem:

What is the wavelength of an earthquake that shakes you with a frequency of 10.0 Hz and gets to another city 84.0 km away in 12.0 s?

Solution:
Equation:

$$\lambda = 700 \text{ m}$$

Exercise:

Problem:

Radio waves transmitted through space at 3.00×10^{8} m/s by the Voyager spacecraft have a wavelength of 0.120 m. What is their frequency?

Exercise:

Problem:

Your ear is capable of differentiating sounds that arrive at the ear just 1.00 ms apart. What is the minimum distance between two speakers that produce sounds that arrive at noticeably different times on a day when the speed of sound is 340 m/s?

Solution:
Equation:

$$d=34.0\text{ cm}$$

Exercise:

Problem:

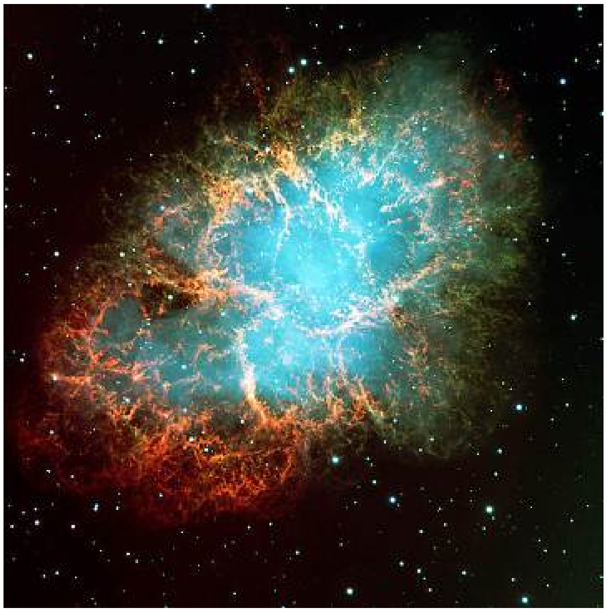
(a) Seismographs measure the arrival times of earthquakes with a precision of 0.100 s. To get the distance to the epicenter of the quake, they compare the arrival times of S- and P-waves, which travel at different speeds. [link]) If S- and P-waves travel at 4.00 and 7.20 km/s, respectively, in the region considered, how precisely can the distance to the source of the earthquake be determined? (b) Seismic waves from underground detonations of nuclear bombs can be used to locate the test site and detect violations of test bans. Discuss whether your answer to (a) implies a serious limit to such detection. (Note also that the uncertainty is greater if there is an uncertainty in the propagation speeds of the S- and P-waves.)

A seismograph as described in above problem.(credit: Oleg Alexandrov)

Exercise:

Problem:

The Crab Nebula (see [link]) pulsar is the remnant of a supernova that occurred in A.D. 1054. Using data from [link], calculate the approximate factor by which the power output of this astronomical object has declined since its explosion.



Crab Nebula (credit: ESO, via Wikimedia Commons)

Solution:
Equation:

$$2\times10^{-10}$$

Exercise:

Problem:

Suppose a star 1000 times brighter than our Sun (that is, emitting 1000 times the power) suddenly goes supernova. Using data from [link]: (a) By what factor does its power output increase? (b) How many times brighter than our entire Milky Way galaxy is the supernova? (c) Based on your answers, discuss whether it should be possible to observe supernovas in distant galaxies. Note that there are on the order of 10^{11} observable galaxies, the average brightness of which is somewhat less than our own galaxy.

Exercise:

Problem:

A person in good physical condition can put out 100 W of useful power for several hours at a stretch, perhaps by pedaling a mechanism that drives an electric generator. Neglecting any problems of generator efficiency and practical considerations such as resting time: (a) How many people would it take to run a 4.00-kW electric clothes dryer? (b) How many people would it take to replace a large electric power plant that generates 800 MW?

Solution:

(a) 40

(b) 8 million

Exercise:

Problem:

What is the cost of operating a 3.00-W electric clock for a year if the cost of electricity is $0.0900 per $\mathrm{kW \cdot h}$?

Exercise:

Problem:

A large household air conditioner may consume 15.0 kW of power. What is the cost of operating this air conditioner 3.00 h per day for 30.0 d if the cost of electricity is $0.110 per $\mathrm{kW \cdot h}$?

Solution:

$149

Exercise:

Problem:

(a) What is the average power consumption in watts of an appliance that uses $5.00 \text{ kW} \cdot \text{h}$ of energy per day? (b) How many joules of energy does this appliance consume in a year?

Exercise:

Problem:

(a) What is the average useful power output of a person who does 6.00×10^6 J of useful work in 8.00 h? (b) Working at this rate, how long will it take this person to lift 2000 kg of bricks 1.50 m to a platform? (Work done to lift his body can be omitted because it is not considered useful output here.)

Solution:

(a) 208 W

(b) 141 s

Exercise:

Problem:

A 500-kg dragster accelerates from rest to a final speed of 110 m/s in 400 m (about a quarter of a mile) and encounters an average frictional force of 1200 N. What is its average power output in watts and horsepower if this takes 7.30 s?

Exercise:

Problem:

(a) How long will it take an 850-kg car with a useful power output of 40.0 hp (1 hp = 746 W) to reach a speed of 15.0 m/s, neglecting friction? (b) How long will this acceleration take if the car also climbs a 3.00-m-high hill in the process?

Solution:

(a) 3.20 s

(b) 4.04 s

Exercise:

Problem:

(a) Find the useful power output of an elevator motor that lifts a 2500-kg load a height of 35.0 m in 12.0 s, if it also increases the speed from rest to 4.00 m/s. Note that the total mass of the counterbalanced system is 10,000 kg—so that only 2500 kg is raised in height, but the full 10,000 kg is accelerated. (b) What does it cost, if electricity is $0.0900 per $\mathrm{kW \cdot h}$?

Exercise:

Problem:

(a) What is the available energy content, in joules, of a battery that operates a 2.00-W electric clock for 18 months? (b) How long can a battery that can supply $8.00 \times 10^4\ \mathrm{J}$ run a pocket calculator that consumes energy at the rate of $1.00 \times 10^{-3}\ \mathrm{W}$?

Solution:

(a) $9.46 \times 10^7\ \mathrm{J}$

(b) $2.54\ \mathrm{y}$

Exercise:

Problem:

(a) How long would it take a 1.50×10^5-kg airplane with engines that produce 100 MW of power to reach a speed of 250 m/s and an altitude of 12.0 km if air resistance were negligible? (b) If it actually takes 900 s, what is the power? (c) Given this power, what is the average force of air resistance if the airplane takes 1200 s? (Hint: You must find the distance the plane travels in 1200 s assuming constant acceleration.)

Exercise:

Problem:

Calculate the power output needed for a 950-kg car to climb a 2.00^{o} slope at a constant 30.0 m/s while encountering wind resistance and friction totaling 600 N. Explicitly show how you follow the steps in the Problem-Solving Strategies for Energy.

Solution:

Identify knowns: $m = 950\text{ kg}$, slope angle $\theta = 2.00^{\mathrm{o}}$, $v = 3.00\text{ m/s}$, $f = 600\text{ N}$

Identify unknowns: power P of the car, force F that car applies to road

Solve for unknown:

$$P = \frac{W}{t} = \frac{\text{Fd}}{t} = F\left(\frac{d}{t}\right) = \text{Fv},$$

where F is parallel to the incline and must oppose the resistive forces and the force of gravity:

$$F = f + w = 600\text{ N} + \text{mg sin }\theta$$

Insert this into the expression for power and solve:

$$\begin{aligned} P &= (f + \text{mg sin }\theta)v \\ &= \left[600\text{ N} + (950\text{ kg})\left(9.80\text{ m/s}^2\right)\text{sin }2^{\mathrm{o}}\right](30.0\text{ m/s}) \\ &= 2.77 \times 10^4\text{ W} \end{aligned}$$

About 28 kW (or about 37 hp) is reasonable for a car to climb a gentle incline.

Exercise:

Problem:

(a) Calculate the power per square meter reaching Earth's upper atmosphere from the Sun. (Take the power output of the Sun to be $4.00 \times 10^{26}\ \mathrm{W}$.) (b) Part of this is absorbed and reflected by the atmosphere, so that a maximum of $1.30\ \mathrm{kW/m^2}$ reaches Earth's surface. Calculate the area in km^2 of solar energy collectors needed to replace an electric power plant that generates 750 MW if the collectors convert an average of 2.00% of the maximum power into electricity. (This small conversion efficiency is due to the devices themselves, and the fact that the sun is directly overhead only briefly.) With the same assumptions, what area would be needed to meet the United States' energy needs $(1.05 \times 10^{20}\ \mathrm{J})$? Australia's energy needs $(5.4 \times 10^{18}\ \mathrm{J})$? China's energy needs $(6.3 \times 10^{19}\ \mathrm{J})$? (These energy consumption values are from 2006.)

Exercise:

Problem: Engineering Application

Each piston of an engine makes a sharp sound every other revolution of the engine. (a) How fast is a race car going if its eight-cylinder engine emits a sound of frequency 750 Hz, given that the engine makes 2000 revolutions per kilometer? (b) At how many revolutions per minute is the engine rotating?

Solution:

(a) 93.8 m/s

(b) $11.3 \times 10^3\ \mathrm{rev/min}$

Exercise:

Problem: Medical Application

Ultrasound of intensity $1.50 \times 10^2\ \mathrm{W/m^2}$ is produced by the rectangular head of a medical imaging device measuring 3.00 by 5.00 cm. What is its power output?

Solution:

0.225 W

Exercise:

Problem:

The low-frequency speaker of a stereo set has a surface area of $0.05\ \mathrm{m^2}$ and produces 1W of acoustical power. What is the intensity at the speaker? If the speaker projects sound uniformly in all directions, at what distance from the speaker is the intensity $0.1\ \mathrm{W/m^2}$?

Exercise:

Problem:

To increase intensity of a wave by a factor of 50, by what factor should the amplitude be increased?

Solution:

7.07

Exercise:

Problem: **Engineering Application**

A device called an insolation meter is used to measure the intensity of sunlight has an area of 100 cm^2 and registers 6.50 W. What is the intensity in $\mathrm{W/m^2}$?

Exercise:

Problem: **Astronomy Application**

Energy from the Sun arrives at the top of the Earth’s atmosphere with an intensity of $1.30\ \mathrm{kW/m^2}$. How long does it take for $1.8 \times 10^9\ \mathrm{J}$ to arrive on an area of $1.00\ \mathrm{m^2}$?

Solution:

16.0 d

Exercise:

Problem:

Suppose you have a device that extracts energy from ocean breakers in direct proportion to their intensity. If the device produces 10.0 kW of power on a day when the breakers are 1.20 m high, how much will it produce when they are 0.600 m high?

Solution:

2.50 kW

Exercise:

Problem: Engineering Application

(a) A photovoltaic array of (solar cells) is 10.0% efficient in gathering solar energy and converting it to electricity. If the average intensity of sunlight on one day is $700\ \mathrm{W/m^2}$, what area should your array have to gather energy at the rate of 100 W? (b) What is the maximum cost of the array if it must pay for itself in two years of operation averaging 10.0 hours per day? Assume that it earns money at the rate of 9.00 ¢ per kilowatt-hour.

Exercise:

Problem:

A microphone receiving a pure sound tone feeds an oscilloscope, producing a wave on its screen. If the sound intensity is originally $2.00 \times 10^{-5}\ \mathrm{W/m^2}$, but is turned up until the amplitude increases by 30.0%, what is the new intensity?

Solution:

$3.38 \times 10^{-5}\ \mathrm{W/m^2}$

Exercise:

Problem: **Medical Application**

(a) What is the intensity in $\mathrm{W/m^2}$ of a laser beam used to burn away cancerous tissue that, when 90.0% absorbed, puts 500 J of energy into a circular spot 2.00 mm in diameter in 4.00 s? (b) Discuss how this intensity compares to the average intensity of sunlight (about $700\ \mathrm{W/m^2}$) and the implications that would have if the laser beam entered your eye. Note how your answer depends on the time duration of the exposure.

Glossary

intensity
: power per unit area

Where does Light Come From
Exercise:

UMASS AMHERST Instructor's Notes

Problem:

This section is also available as a video on the UMass Physics 13X YouTube page. The link can be found here.

Where does light come from?

Light is generated any time a charge undergoes acceleration, this is a connection to an idea from Physics 131. Just like in Physics 131 it's not the motion of the charge that matters, but its acceleration. Moving charges don't generate light only accelerating ones do. To expand upon this connection to 131 a little bit more, if a charge accelerates by slowing down, it is still accelerating then from Newton's second law,

$$F = ma$$

, we know that a force has acted upon it. Assuming that it takes some distance for this slowing down to occur, then the force must have been applied for some distance and we know that work was therefore done on the charged particle. By the statement of conservation of energy, or equivalently the first law of thermodynamics, if work is done on a particle then the particles energy must change, that energy must go somewhere and where does it often go? It goes into light.

Here’s an example with which you might be familiar from your chemistry class. An electron in an outer energy level of an atom falls to a lower energy level. There's a change in energy as the electron falls, that energy has to go somewhere. It goes into the release of light.

Electrons changing energy levels, however, is not the only way to produce light. Think about an old-school incandescent lamp with the filament in it

that get hot as you turn them on, to understand why these incandescent lights give off light we have to understand a little bit about what temperature is.

Recall from Physics 131 that temperature is related to the average kinetic energies of particles moving around randomly on the atomic and subatomic scales. As these particles are bouncing around randomly, they're changing directions. From 131 we know that acceleration is a vector, so because velocity changes direction, then we know that there is acceleration. So once again, even any object with temperature will emit light due to the accelerating charges bouncing around on the atomic and subatomic scale.
Exercise:

UMASS AMHERST Instructor's Notes

Problem:
In Summary:

- Every object with a temperature (i.e. everything) will emit some amount of light of some type.
- Our eyes, however, are only sensitive to certain kinds of light and we therefore cannot see this light from everyday objects such as you and I. We don't see light coming off of us because our eyes are not sensitive to the kind of light that we emit due to our temperature.
- However, we can build devices that can see the light given off by more everyday objects such as people by using technologies such as infrared cameras.

Properties of Light
Exercise:

UMASS AMHERST Instructor's Notes

Problem:

This section is also available as a video on the UMass Physics 13X YouTube page. The link can be found here.

What are some properties of light waves?

Like all waves, light waves are characterized by a wavelength, a frequency, a speed, which follows the usual relationship of

$$v=\lambda f$$

, and an amplitude. However, there are some important unique characteristics of light waves. For light, the wave in the vacuum speed is always the same,

$$c=3\times 10^8$$

meters per second. In a vacuum

$$v=\lambda f$$

, turns into

$$c=\lambda f$$

, because all light waves, regardless of their wavelength or frequency or amplitude, travel at this same fundamental speed.

For the amplitude of the light wave we will not use the symbol

$$A$$

we will instead use the symbol

$$E$$

and the amplitude of a light wave has the units of Newton's per Coulomb, Newton's are the unit of force and Coulomb, as you've already discussed elsewhere in your prep, is the unit of a charge. The amplitude of a light wave is a Newton per Coulomb we will see why this is the unit of a light waves amplitude later in this particular course, but for right now you just need to know that those are the units.

There are many different kinds of light. Where do these different kinds of light come from? Well different wavelengths or frequencies represent different kinds of light. Light is also sometimes called electromagnetic radiation, and so the kinds of light are called the E/M spectrum. You'll see the terms ‘electromagnetic spectrum’ or ‘E/M spectrum’ used, which just means the kinds of light. You'll explore more of the different kinds of light in the next section.

But this is giving you a bit of a hint on where this whole course is going and how light, electricity, and magnetism are all going to be deeply connected in some fundamental way, which will come to by the end of this course.

We've now seen that the frequency or wavelength of a light wave tells us what kind of light we are going to have. What does the amplitude of the light wave correspond to?

The amplitude, remember we're using

$$E$$

for the amplitude, is related to the intensity of the light, as in the watts per square meter, by this expression
Equation:

$$I = \frac{1}{2} c \varepsilon_0 E^2,$$

$$c$$

is the usual speed of light

$$3 \times 10^{8}$$

meters per second and

$$\varepsilon_0$$

is a property of just empty space, you might not think of empty space as having properties, but it does,

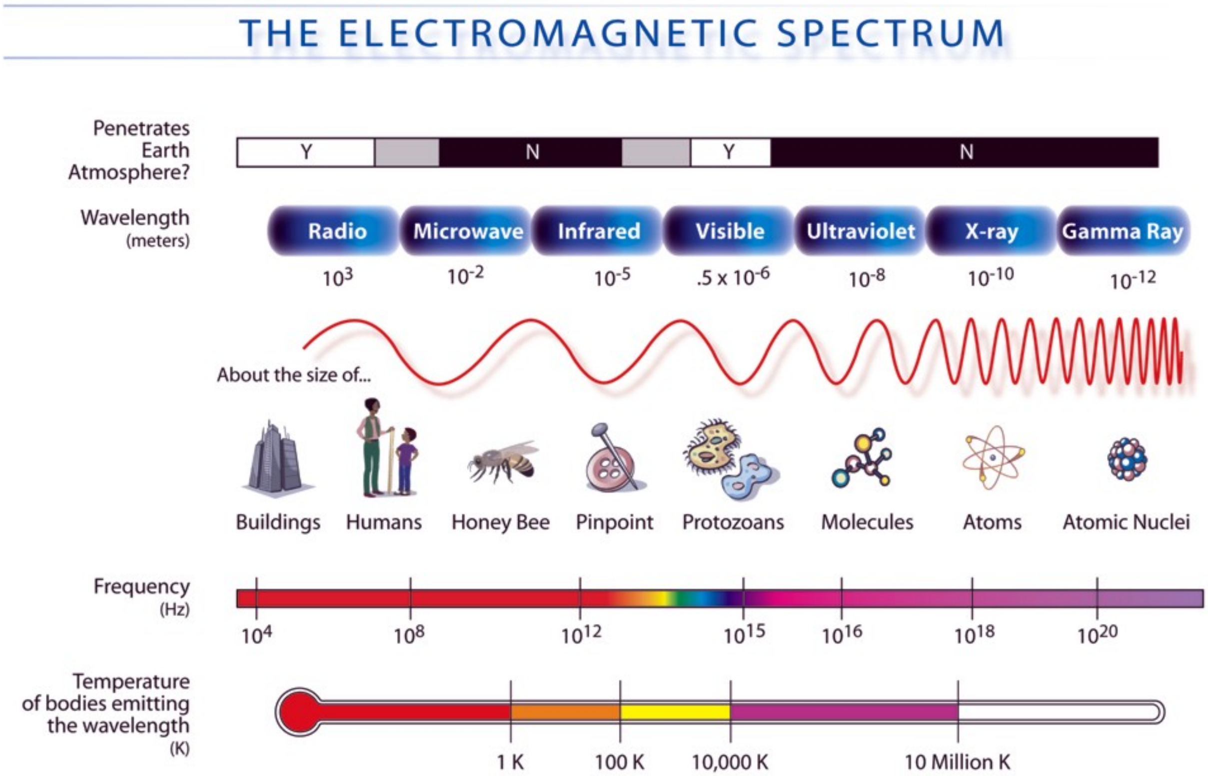
$$\varepsilon_0$$

is a property of empty space called the permittivity of free space, and it has this value

$$8.85 \times 10^{-12} \frac{C^2}{J \bullet m}$$

. We will talk more about this number throughout this course, for now, you just need to know it's a property of empty space.

The Main Parts of the Electromagnetic Spectrum



The electromagnetic spectrum (Credit: NASA)

Exercise:

UMASS AMHERST Instructor's Notes

Problem:

As scientifically trained people, you should have a basic familiarity with the electromagnetic spectrum. Thus, while this course is generally not about memorization, I will ask you to memorize the large basic divisions of the electromagnetic spectrum: radio, microwaves, infrared, visible, ultraviolet, x-rays, and gamma rays. You need to know that radio represents the longest wavelength and gamma rays represent the shortest wavelength. You should also know that, within visible light, red is the longest going through the rainbow to violet. You do NOT need to know the frequencies or wavelengths corresponding to each range. The only exception to this rule is that I do expect you to know that red is about 700nm wavelength while violet is about 350nm. The different types of radiation come up so frequently in scientific discussion that it is important to know some basic facts.

Introduction to the Photon
Exercise:

UMASS AMHERST Instructor's Notes

Problem:

This section is also available as a video on the [UMass Physics 13X YouTube page.](#) The link can be found [here.](#)

We've talked about light as a wave, we've talked about its frequency, its wavelength, its speed, its amplitude. We've talked about the wave properties of light, now we're going to move and think about the particle properties of light. What happens when we think of light as a particle as opposed to as a wave?

Let’s say we have a laser, can I keep making this laser dot dimmer and dimmer and dimmer forever? This may seem like a very abstract philosophical question. I'm going to flip it on its head for you. Can I take a sample of water and keep reducing its amount forever?

No, eventually I get down to one water molecule, and I'm done. This was the basis for the atomic theory. You can't separate matter forever. I'm just asking you the exact same question for a dot of light, can I keep having it forever? And it turns out the answer is no, I can't. At some point I reach the bottom, there's a smallest dimness, just like there's a smallest amount of water you can have, there's a smallest amount of light you can have, and we call this smallest amount of light we say it's a particle of light, and we call it a photon, and we are going to use this symbol

$$\gamma$$

for photon.

We can think of this laser as a light wave where I change the amplitude to make it brighter or darker, or we can flip that on its head and say it's a

bunch of photons flying along together and to make it brighter or darker I changed the number of photons. Already we're sort of bouncing back and forth between thinking of things as waves and particles. This photon image is really good when we think about light being absorbed by materials or emitted from materials, that's when thinking in terms of particles tends to be a good picture. Waves on the other hand tend to do really well when we're thinking about light flying through space.

Let's go through the properties of the photon. We are now imagining light to be made up of little balls, but we are imagining them to be made up of little massless spheres. Little massless particles that travel at the speed of light,

$$c$$

. But even though they are massless they still carry energy and momentum.

The energy of a single photon is related to the wave of the light. The energy of the photon is related to the frequency of the light wave through this expression

$$E = hf$$

, where

$$h$$

is Planck's constant,

$$6.626 \times 10^{-34} J \bullet s$$

.

Exercise:

UMASS AMHERST Instructor's Notes

Problem:

You never need to memorize the number. I'll give it to you, but you can see the units are energy times time

Photons, in addition to having energy, also have momentum. This is the part that tends to get folks, because in 131 I told you that momentum was mass times velocity which is mostly true. It's true as long as you're not going too fast, once you start getting close to the speed of light this will actually break down on you. You need a new expression. But as long as you're going slow, this is fine. But clearly this does not work for photons because for photons mass is zero. Special relativity has an answer, it's the momentum of a photon is the energy divided by the speed of light,

$$p = \frac{E}{c}$$

.

We're going to start with a single photon of green light green light has a wavelength of

$$500$$

nanometers.

Exercise:

UMASS AMHERST Instructor's Notes

Problem:

That's a number that I'm not going to ask you to memorize, but I suspect by the end of the semester you probably have.

Let's go and try and find the energy of this photon. Well we know that

$$E = hf$$

. That's our energy of a photon, but I don't have

$$f$$

. We have wavelength, and we do know that because it's a light wave

$$v = c$$

, and

$$v = \lambda f$$

, so

$$f = \frac{c}{\lambda}$$

.

Exercise:

UMASS AMHERST Instructor's Notes

Problem:

Notice I have not put in a number yet. This is how you should do problems.

Substituting that into there we get

$$E = \frac{h \bullet c}{\lambda}$$

and now we plug our values in and get an energy of

$$3.98 \times 10^{-19} Joules$$

.

That's an annoyingly ugly number to look at so I'm going to introduce a new unit. The electron volt also called the

$$eV$$

. We will talk about where this unit comes from later, but right now you just need to know that it is a unit of energy,

$$1eV$$

is

$$1.62\times10^{-19}Joules$$

.

Exercise:

UMASS AMHERST Instructor's Notes

Problem: Again, I would give you the conversion.

It's just a tiny amount of energy, just like a calorie is a unit of energy or a Joule is the unit of energy, an

$$eV$$

is a unit of energy. It's a nice unit of energy though because it tends to work well when you get down to atomic sizes. For example, I need

$$13.6eV$$

of energy to remove an electron from a hydrogen atom.

Let's convert this, so we got

$$3.98 \times 10^{-19} Joules$$

, we know

$$1eV$$

is

$$1.62 \times 10^{-19} Joules$$

, and I'm left with an energy of this photon of

$$2.48eV$$

. Then we can move on to the momentum. Where we know that momentum is

$$p = \frac{E}{c}$$

, and we can go back to Joules and we got a momentum of

$$1.3 \times 10^{-27} Joules$$

.

Photon Energies and the Electromagnetic Spectrum

- Explain the relationship between the energy of a photon in joules or electron volts and its wavelength or frequency.
- Calculate the number of photons per second emitted by a monochromatic source of specific wavelength and power.

Ionizing Radiation

A photon is a quantum of EM radiation. Its energy is given by $E = \text{hf}$ and is related to the frequency f and wavelength λ of the radiation by

Equation:

$$E = \text{hf} = \frac{\text{hc}}{\lambda}(\text{energy of a photon}),$$

where E is the energy of a single photon and c is the speed of light. When working with small systems, energy in eV is often useful. Note that Planck’s constant in these units is

Equation:

$$h = 4.14 \times 10^{-15}\text{ eV} \cdot \text{s}.$$

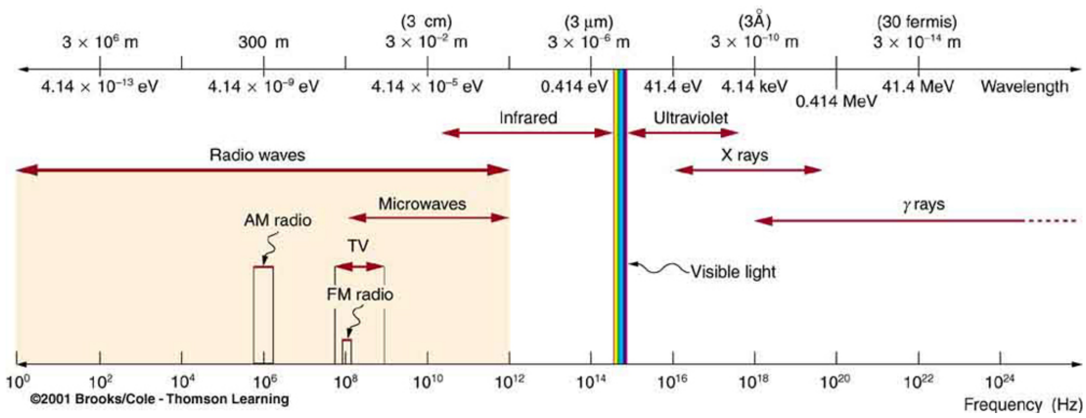
Since many wavelengths are stated in nanometers (nm), it is also useful to know that

Equation:

$$\text{hc} = 1240\text{ eV} \cdot \text{nm}.$$

These will make many calculations a little easier.

All EM radiation is composed of photons. [link] shows various divisions of the EM spectrum plotted against wavelength, frequency, and photon energy. Previously in this book, photon characteristics were alluded to in the discussion of some of the characteristics of UV, x rays, and γ rays, the first of which start with frequencies just above violet in the visible spectrum. It was noted that these types of EM radiation have characteristics much different than visible light. We can now see that such properties arise because photon energy is larger at high frequencies.



The EM spectrum, showing major categories as a function of photon energy in eV, as well as wavelength and frequency. Certain characteristics of EM radiation are directly attributable to photon energy alone.

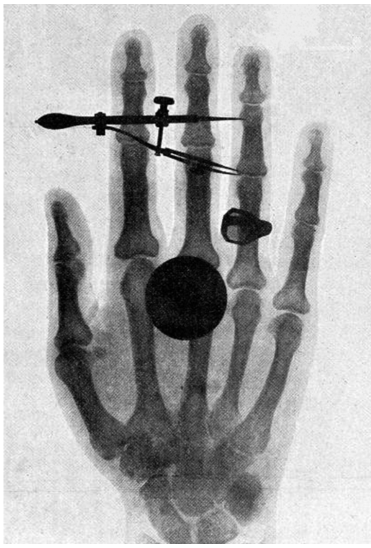
Rotational energies of molecules	10^{-5} eV
Vibrational energies of molecules	0.1 eV
Energy between outer electron shells in atoms	1 eV
Binding energy of a weakly bound molecule	1 eV
Energy of red light	2 eV
Binding energy of a tightly bound molecule	10 eV
Energy to ionize atom or molecule	10 to 1000 eV

Representative Energies for Submicroscopic Effects (Order of Magnitude Only)

Photons act as individual quanta and interact with individual electrons, atoms, molecules, and so on. The energy a photon carries is, thus, crucial to the effects it has. [link] lists representative submicroscopic energies in eV. When we compare photon energies from the EM spectrum in [link] with energies in the table, we can see how effects vary with the type of EM radiation.

Gamma rays, a form of nuclear and cosmic EM radiation, can have the highest frequencies and, hence, the highest photon energies in the EM spectrum. For example, a γ-ray photon with $f = 10^{21}$ Hz has an energy $E = \text{hf} = 6.63 \times 10^{-13}\ \text{J} = 4.14\ \text{MeV}$. This is sufficient energy to ionize thousands of atoms and molecules, since only 10 to 1000 eV are needed per ionization. In fact, γ rays are one type of **ionizing radiation**, as are x rays and UV, because they produce ionization in materials that absorb

them. Because so much ionization can be produced, a single γ-ray photon can cause significant damage to biological tissue, killing cells or damaging their ability to properly reproduce. When cell reproduction is disrupted, the result can be cancer, one of the known effects of exposure to ionizing radiation. Since cancer cells are rapidly reproducing, they are exceptionally sensitive to the disruption produced by ionizing radiation. This means that ionizing radiation has positive uses in cancer treatment as well as risks in producing cancer.



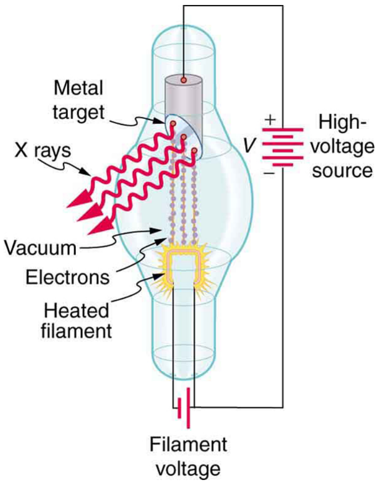
One of the first x-ray images, taken by Röentgen himself. The hand belongs to Bertha Röentgen, his wife. (credit: Wilhelm Conrad Röntgen, via Wikimedia Commons)

High photon energy also enables γ rays to penetrate materials, since a collision with a single atom or molecule is unlikely to absorb all the γ ray's energy. This can make γ rays useful as a probe, and they are sometimes used in medical imaging. **x rays**, as you can see in [link], overlap with the low-frequency end of the γ ray range. Since x rays have energies of keV and up, individual x-ray photons also can produce large amounts of ionization. At lower photon energies, x rays are not as penetrating as γ rays and are slightly less hazardous. X rays are ideal for medical imaging, their most common use, and a fact that was recognized immediately upon their discovery in 1895 by the German physicist W. C. Roentgen (1845–1923). (See [link].) Within one year of their discovery, x rays (for a time called Roentgen rays) were used for medical diagnostics. Roentgen received the 1901 Nobel Prize for the discovery of x rays.

Note:

Connections: Conservation of Energy

Once again, we find that conservation of energy allows us to consider the initial and final forms that energy takes, without having to make detailed calculations of the intermediate steps. [link] is solved by considering only the initial and final forms of energy.



X rays are produced when energetic electrons strike the copper anode of this cathode ray tube (CRT). Electrons (shown here as separate particles) interact individually with the material they strike, sometimes producing photons of EM radiation.

While γ rays originate in nuclear decay, x rays are produced by the process shown in [link]. Electrons ejected by thermal agitation from a hot filament in a vacuum tube are accelerated through a high voltage, gaining kinetic energy from the electrical potential energy. When they strike the anode, the electrons convert their kinetic energy to a variety of forms, including thermal energy. But since an accelerated charge radiates EM waves, and since the electrons act individually, photons are also produced. Some of these x-ray photons obtain the kinetic energy of the electron. The accelerated electrons originate at the cathode, so such a tube is called a cathode ray tube (CRT), and various versions of them are found in older TV and computer screens as well as in x-ray machines.

Example:
X-ray Photon Energy and X-ray Tube Voltage

Find the maximum energy in eV of an x-ray photon produced by electrons accelerated through a potential difference of 50.0 kV in a CRT like the one in [link].

Strategy

Electrons can give all of their kinetic energy to a single photon when they strike the anode of a CRT. (This is something like the photoelectric effect in reverse.) The kinetic energy of the electron comes from electrical potential energy. Thus we can simply equate the maximum photon energy to the electrical potential energy—that is, $hf = qV$. (We do not have to calculate each step from beginning to end if we know that all of the starting energy qV is converted to the final form hf.)

Solution

The maximum photon energy is $\mathrm{hf} = \mathrm{qV}$, where q is the charge of the electron and V is the accelerating voltage. Thus,

Equation:

$$\mathrm{hf} = (1.60 \times 10^{-19}\ \mathrm{C})(50.0 \times 10^{3}\ \mathrm{V}).$$

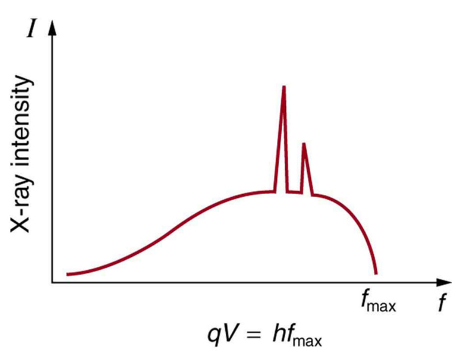
From the definition of the electron volt, we know $1\ \mathrm{eV} = 1.60 \times 10^{-19}\ \mathrm{J}$, where $1\ \mathrm{J} = 1\ \mathrm{C} \cdot \mathrm{V}$. Gathering factors and converting energy to eV yields

Equation:

$$\mathrm{hf} = (50.0 \times 10^{3})(1.60 \times 10^{-19}\ \mathrm{C} \cdot \mathrm{V})\left(\frac{1\ \mathrm{eV}}{1.60 \times 10^{-19}\ \mathrm{C} \cdot \mathrm{V}}\right) = (50.0 \times 10^{3})(1\ \mathrm{eV}) = 50.0\ \mathrm{keV}.$$

Discussion

This example produces a result that can be applied to many similar situations. If you accelerate a single elementary charge, like that of an electron, through a potential given in volts, then its energy in eV has the same numerical value. Thus a 50.0-kV potential generates 50.0 keV electrons, which in turn can produce photons with a maximum energy of 50 keV. Similarly, a 100-kV potential in an x-ray tube can generate up to 100-keV x-ray photons. Many x-ray tubes have adjustable voltages so that various energy x rays with differing energies, and therefore differing abilities to penetrate, can be generated.



X-ray spectrum obtained when energetic electrons strike a material. The smooth part of the spectrum is bremsstrahlung, while the peaks are characteristic of the anode material. Both are atomic processes that produce energetic

photons known as x-ray photons.

[link] shows the spectrum of x rays obtained from an x-ray tube. There are two distinct features to the spectrum. First, the smooth distribution results from electrons being decelerated in the anode material. A curve like this is obtained by detecting many photons, and it is apparent that the maximum energy is unlikely. This decelerating process produces radiation that is called **bremsstrahlung** (German for *braking radiation*). The second feature is the existence of sharp peaks in the spectrum; these are called **characteristic x rays**, since they are characteristic of the anode material. Characteristic x rays come from atomic excitations unique to a given type of anode material. They are akin to lines in atomic spectra, implying the energy levels of atoms are quantized. Phenomena such as discrete atomic spectra and characteristic x rays are explored further in Atomic Physics.

Ultraviolet radiation (approximately 4 eV to 300 eV) overlaps with the low end of the energy range of x rays, but UV is typically lower in energy. UV comes from the de-excitation of atoms that may be part of a hot solid or gas. These atoms can be given energy that they later release as UV by numerous processes, including electric discharge, nuclear explosion, thermal agitation, and exposure to x rays. A UV photon has sufficient energy to ionize atoms and molecules, which makes its effects different from those of visible light. UV thus has some of the same biological effects as γ rays and x rays. For example, it can cause skin cancer and is used as a sterilizer. The major difference is that several UV photons are required to disrupt cell reproduction or kill a bacterium, whereas single γ-ray and X-ray photons can do the same damage. But since UV does have the energy to alter molecules, it can do what visible light cannot. One of the beneficial aspects of UV is that it triggers the production of vitamin D in the skin, whereas visible light has insufficient energy per photon to alter the molecules that trigger this production. Infantile jaundice is treated by exposing the baby to UV (with eye protection), called phototherapy, the beneficial effects of which are thought to be related to its ability to help prevent the buildup of potentially toxic bilirubin in the blood.

Example:
Photon Energy and Effects for UV
Short-wavelength UV is sometimes called vacuum UV, because it is strongly absorbed by air and must be studied in a vacuum. Calculate the photon energy in eV for 100-nm vacuum UV, and estimate the number of molecules it could ionize or break apart.
Strategy
Using the equation $E = \text{hf}$ and appropriate constants, we can find the photon energy and compare it with energy information in [link].
Solution
The energy of a photon is given by
Equation:

$$E = \text{hf} = \frac{\text{hc}}{\lambda}.$$

Using $\text{hc} = 1240\ \text{eV} \cdot \text{nm}$, we find that
Equation:

$$E = \frac{\text{hc}}{\lambda} = \frac{1240 \text{ eV} \cdot \text{nm}}{100 \text{ nm}} = 12.4 \text{ eV}.$$

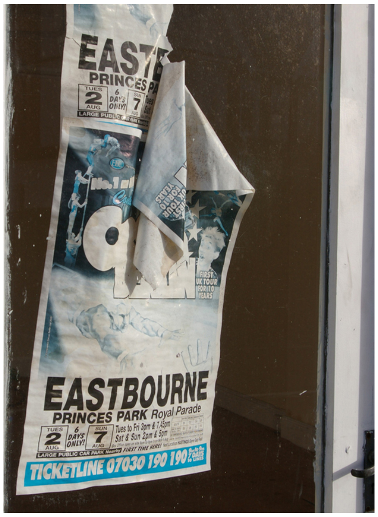
Discussion

According to [link], this photon energy might be able to ionize an atom or molecule, and it is about what is needed to break up a tightly bound molecule, since they are bound by approximately 10 eV. This photon energy could destroy about a dozen weakly bound molecules. Because of its high photon energy, UV disrupts atoms and molecules it interacts with. One good consequence is that all but the longest-wavelength UV is strongly absorbed and is easily blocked by sunglasses. In fact, most of the Sun's UV is absorbed by a thin layer of ozone in the upper atmosphere, protecting sensitive organisms on Earth. Damage to our ozone layer by the addition of such chemicals as CFC's has reduced this protection for us.

Visible Light

The range of photon energies for **visible light** from red to violet is 1.63 to 3.26 eV, respectively (left for this chapter's Problems and Exercises to verify). These energies are on the order of those between outer electron shells in atoms and molecules. This means that these photons can be absorbed by atoms and molecules. A *single* photon can actually stimulate the retina, for example, by altering a receptor molecule that then triggers a nerve impulse. Photons can be absorbed or emitted only by atoms and molecules that have precisely the correct quantized energy step to do so. For example, if a red photon of frequency f encounters a molecule that has an energy step, ΔE, equal to hf, then the photon can be absorbed. Violet flowers absorb red and reflect violet; this implies there is no energy step between levels in the receptor molecule equal to the violet photon's energy, but there is an energy step for the red.

There are some noticeable differences in the characteristics of light between the two ends of the visible spectrum that are due to photon energies. Red light has insufficient photon energy to expose most black-and-white film, and it is thus used to illuminate darkrooms where such film is developed. Since violet light has a higher photon energy, dyes that absorb violet tend to fade more quickly than those that do not. (See [link].) Take a look at some faded color posters in a storefront some time, and you will notice that the blues and violets are the last to fade. This is because other dyes, such as red and green dyes, absorb blue and violet photons, the higher energies of which break up their weakly bound molecules. (Complex molecules such as those in dyes and DNA tend to be weakly bound.) Blue and violet dyes reflect those colors and, therefore, do not absorb these more energetic photons, thus suffering less molecular damage.



Why do the reds, yellows, and greens fade before the blues and violets when exposed to the Sun, as with this poster? The answer is related to photon energy. (credit: Deb Collins, Flickr)

Transparent materials, such as some glasses, do not absorb any visible light, because there is no energy step in the atoms or molecules that could absorb the light. Since individual photons interact with individual atoms, it is nearly impossible to have two photons absorbed simultaneously to reach a large energy step. Because of its lower photon energy, visible light can sometimes pass through many kilometers of a substance, while higher frequencies like UV, x ray, and γ rays are absorbed, because they have sufficient photon energy to ionize the material.

Example:
How Many Photons per Second Does a Typical Light Bulb Produce?
Assuming that 10.0% of a 100-W light bulb's energy output is in the visible range (typical for incandescent bulbs) with an average wavelength of 580 nm, calculate the number of visible photons emitted per second.
Strategy
Power is energy per unit time, and so if we can find the energy per photon, we can determine the number of photons per second. This will best be done in joules, since power is given in watts, which are joules per second.
Solution
The power in visible light production is 10.0% of 100 W, or 10.0 J/s. The energy of the average visible photon is found by substituting the given average wavelength into the formula
Equation:

$$E = \frac{\text{hc}}{\lambda}.$$

This produces

Equation:

$$E = \frac{(6.63 \times 10^{-34}\ \text{J} \cdot \text{s})(3.00 \times 10^{8}\ \text{m/s})}{580 \times 10^{-9}\ \text{m}} = 3.43 \times 10^{-19}\ \text{J}.$$

The number of visible photons per second is thus

Equation:

$$\text{photon/s} = \frac{10.0\ \text{J/s}}{3.43 \times 10^{-19}\ \text{J/photon}} = 2.92 \times 10^{19}\ \text{photon/s}.$$

Discussion

This incredible number of photons per second is verification that individual photons are insignificant in ordinary human experience. It is also a verification of the correspondence principle—on the macroscopic scale, quantization becomes essentially continuous or classical. Finally, there are so many photons emitted by a 100-W lightbulb that it can be seen by the unaided eye many kilometers away.

Lower-Energy Photons

Infrared radiation (IR) has even lower photon energies than visible light and cannot significantly alter atoms and molecules. IR can be absorbed and emitted by atoms and molecules, particularly between closely spaced states. IR is extremely strongly absorbed by water, for example, because water molecules have many states separated by energies on the order of 10^{-5} eV to 10^{-2} eV, well within the IR and microwave energy ranges. This is why in the IR range, skin is almost jet black, with an emissivity near 1—there are many states in water molecules in the skin that can absorb a large range of IR photon energies. Not all molecules have this property. Air, for example, is nearly transparent to many IR frequencies.

Microwaves are the highest frequencies that can be produced by electronic circuits, although they are also produced naturally. Thus microwaves are similar to IR but do not extend to as high frequencies. There are states in water and other molecules that have the same frequency and energy as microwaves, typically about 10^{-5} eV. This is one reason why food absorbs microwaves more strongly than many other materials, making microwave ovens an efficient way of putting energy directly into food.

Photon energies for both IR and microwaves are so low that huge numbers of photons are involved in any significant energy transfer by IR or microwaves (such as warming yourself with a heat lamp or cooking pizza in the microwave). Visible light, IR, microwaves, and all lower frequencies cannot produce ionization with single photons and do not ordinarily have the hazards of higher frequencies. When visible, IR, or microwave radiation *is* hazardous, such as the inducement of cataracts by microwaves, the hazard is due to huge numbers of photons acting together (not to an accumulation of photons, such as sterilization by weak UV). The negative effects of visible, IR, or microwave radiation can be thermal effects, which could be produced by any heat source. But one difference is that at very high intensity, strong electric and magnetic fields can be produced by photons acting together. Such electromagnetic fields (EMF) can actually ionize materials.

Note:
Misconception Alert: High-Voltage Power Lines
Although some people think that living near high-voltage power lines is hazardous to one's health, ongoing studies of the transient field effects produced by these lines show their strengths to be insufficient to cause damage. Demographic studies also fail to show significant correlation of ill effects with high-voltage power lines. The American Physical Society issued a report over 10 years ago on power-line fields, which concluded that the scientific literature and reviews of panels show no consistent, significant link between cancer and power-line fields. They also felt that the "diversion of resources to eliminate a threat which has no persuasive scientific basis is disturbing."

It is virtually impossible to detect individual photons having frequencies below microwave frequencies, because of their low photon energy. But the photons are there. A continuous EM wave can be modeled as photons. At low frequencies, EM waves are generally treated as time- and position-varying electric and magnetic fields with no discernible quantization. This is another example of the correspondence principle in situations involving huge numbers of photons.

Note:
PhET Explorations: Color Vision
Make a whole rainbow by mixing red, green, and blue light. Change the wavelength of a monochromatic beam or filter white light. View the light as a solid beam, or see the individual photons.

https://phet.colorado.edu/sims/html/color-vision/latest/color-vision_en.html

Section Summary

- Photon energy is responsible for many characteristics of EM radiation, being particularly noticeable at high frequencies.
- Photons have both wave and particle characteristics.

Glossary

gamma ray
: also γ-ray; highest-energy photon in the EM spectrum

ionizing radiation
: radiation that ionizes materials that absorb it

x ray
: EM photon between γ-ray and UV in energy

bremsstrahlung
: German for *braking radiation*; produced when electrons are decelerated

characteristic x rays

x rays whose energy depends on the material they were produced in

ultraviolet radiation
UV; ionizing photons slightly more energetic than violet light

visible light
the range of photon energies the human eye can detect

infrared radiation
photons with energies slightly less than red light

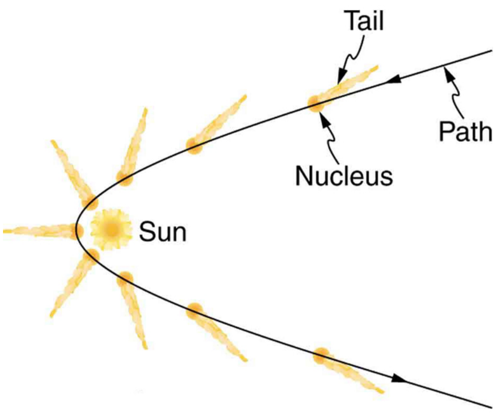
microwaves
photons with wavelengths on the order of a micron (μm)

Photon Momentum

- Relate the linear momentum of a photon to its energy or wavelength, and apply linear momentum conservation to simple processes involving the emission, absorption, or reflection of photons.
- Account qualitatively for the increase of photon wavelength that is observed, and explain the significance of the Compton wavelength.

Measuring Photon Momentum

The quantum of EM radiation we call a **photon** has properties analogous to those of particles we can see, such as grains of sand. A photon interacts as a unit in collisions or when absorbed, rather than as an extensive wave. Massive quanta, like electrons, also act like macroscopic particles—something we expect, because they are the smallest units of matter. Particles carry momentum as well as energy. Despite photons having no mass, there has long been evidence that EM radiation carries momentum. (Maxwell and others who studied EM waves predicted that they would carry momentum.) It is now a well-established fact that photons *do* have momentum. In fact, photon momentum is suggested by the photoelectric effect, where photons knock electrons out of a substance. [link] shows macroscopic evidence of photon momentum.



The tails of the Hale-Bopp comet point away from the Sun, evidence that light has momentum. Dust emanating from the body of the comet forms this tail. Particles of dust are pushed away from the Sun by light reflecting from them. The blue ionized gas tail is also produced by photons interacting with atoms in the comet material. (credit: Geoff Chester, U.S. Navy, via Wikimedia Commons)

[link] shows a comet with two prominent tails. What most people do not know about the tails is that they always point *away* from the Sun rather than trailing behind the comet (like the tail of Bo Peep's sheep). Comet tails are composed of gases and dust

evaporated from the body of the comet and ionized gas. The dust particles recoil away from the Sun when photons scatter from them. Evidently, photons carry momentum in the direction of their motion (away from the Sun), and some of this momentum is transferred to dust particles in collisions. Gas atoms and molecules in the blue tail are most affected by other particles of radiation, such as protons and electrons emanating from the Sun, rather than by the momentum of photons.

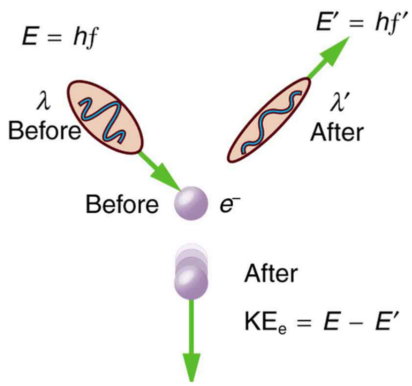
Note:
Connections: Conservation of Momentum
Not only is momentum conserved in all realms of physics, but all types of particles are found to have momentum. We expect particles with mass to have momentum, but now we see that massless particles including photons also carry momentum.

Momentum is conserved in quantum mechanics just as it is in relativity and classical physics. Some of the earliest direct experimental evidence of this came from scattering of x-ray photons by electrons in substances, named Compton scattering after the American physicist Arthur H. Compton (1892–1962). Around 1923, Compton observed that x rays scattered from materials had a decreased energy and correctly analyzed this as being due to the scattering of photons from electrons. This phenomenon could be handled as a collision between two particles—a photon and an electron at rest in the material. Energy and momentum are conserved in the collision. (See [link]) He won a Nobel Prize in 1929 for the discovery of this scattering, now called the **Compton effect**, because it helped prove that **photon momentum** is given by

Equation:

$$p = \frac{h}{\lambda},$$

where h is Planck’s constant and λ is the photon wavelength. (Note that relativistic momentum given as $p = \gamma \text{mu}$ is valid only for particles having mass.)



The Compton effect is the name given to the scattering of a photon by an electron. Energy and momentum are conserved, resulting in a reduction of both for the scattered photon. Studying this effect, Compton verified that photons have momentum.

We can see that photon momentum is small, since $p = h/\lambda$ and h is very small. It is for this reason that we do not ordinarily observe photon momentum. Our mirrors do not recoil when light reflects from them (except perhaps in cartoons). Compton saw the effects of photon momentum because he was observing x rays, which have a small wavelength and a relatively large momentum, interacting with the lightest of particles, the electron.

Example:
Electron and Photon Momentum Compared
(a) Calculate the momentum of a visible photon that has a wavelength of 500 nm. (b) Find the velocity of an electron having the same momentum. (c) What is the energy of the electron, and how does it compare with the energy of the photon?
Strategy

Finding the photon momentum is a straightforward application of its definition: $p = \frac{h}{\lambda}$. If we find the photon momentum is small, then we can assume that an electron with the same momentum will be nonrelativistic, making it easy to find its velocity and kinetic energy from the classical formulas.

Solution for (a)

Photon momentum is given by the equation:

Equation:

$$p = \frac{h}{\lambda}.$$

Entering the given photon wavelength yields

Equation:

$$p = \frac{6.63 \times 10^{-34}\ \text{J} \cdot \text{s}}{500 \times 10^{-9}\ \text{m}} = 1.33 \times 10^{-27}\ \text{kg} \cdot \text{m/s}.$$

Solution for (b)

Since this momentum is indeed small, we will use the classical expression $p = \text{mv}$ to find the velocity of an electron with this momentum. Solving for v and using the known value for the mass of an electron gives

Equation:

$$v = \frac{p}{m} = \frac{1.33 \times 10^{-27}\ \text{kg} \cdot \text{m/s}}{9.11 \times 10^{-31}\ \text{kg}} = 1460\ \text{m/s} \approx 1460\ \text{m/s}.$$

Solution for (c)

The electron has kinetic energy, which is classically given by

Equation:

$$\text{KE}_e = \frac{1}{2} m v^2.$$

Thus,

Equation:

$$\text{KE}_e = \frac{1}{2}(9.11 \times 10^{-3}\ \text{kg})(1455\ \text{m/s})^2 = 9.64 \times 10^{-25}\ \text{J}.$$

Converting this to eV by multiplying by $(1\ \text{eV})/(1.602 \times 10^{-19}\ \text{J})$ yields

Equation:

$$\text{KE}_e = 6.02 \times 10^{-6}\ \text{eV}.$$

The photon energy E is

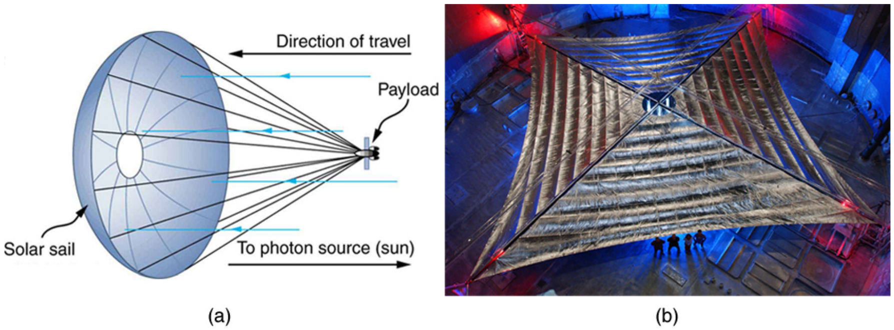
Equation:

$$E = \frac{\text{hc}}{\lambda} = \frac{1240\ \text{eV} \cdot \text{nm}}{500\ \text{nm}} = 2.48\ \text{eV},$$

which is about five orders of magnitude greater.

Discussion

Photon momentum is indeed small. Even if we have huge numbers of them, the total momentum they carry is small. An electron with the same momentum has a 1460 m/s velocity, which is clearly nonrelativistic. A more massive particle with the same momentum would have an even smaller velocity. This is borne out by the fact that it takes far less energy to give an electron the same momentum as a photon. But on a quantum-mechanical scale, especially for high-energy photons interacting with small masses, photon momentum is significant. Even on a large scale, photon momentum can have an effect if there are enough of them and if there is nothing to prevent the slow recoil of matter. Comet tails are one example, but there are also proposals to build space sails that use huge low-mass mirrors (made of aluminized Mylar) to reflect sunlight. In the vacuum of space, the mirrors would gradually recoil and could actually take spacecraft from place to place in the solar system. (See [link].)



(a) Space sails have been proposed that use the momentum of sunlight reflecting from gigantic low-mass sails to propel spacecraft about the solar system. A Russian test model of this (the Cosmos 1) was launched in 2005, but did not make it into orbit due to a rocket failure. (b) A U.S. version of this, labeled LightSail-1, is scheduled for trial launches in the first part of this

decade. It will have a 40-m^2 sail. (credit: Kim Newton/NASA)

Relativistic Photon Momentum

There is a relationship between photon momentum p and photon energy E that is consistent with the relation given previously for the relativistic total energy of a particle as $E^2 = (\text{pc})^2 + (\text{mc})^2$. We know m is zero for a photon, but p is not, so that $E^2 = (\text{pc})^2 + (\text{mc})^2$ becomes

Equation:

$$E = \text{pc},$$

or

Equation:

$$p = \frac{E}{c}\ (\text{photons}).$$

To check the validity of this relation, note that $E = \text{hc}/\lambda$ for a photon. Substituting this into $p = E/c$ yields

Equation:

$$p = (\text{hc}/\lambda)/c = \frac{h}{\lambda},$$

as determined experimentally and discussed above. Thus, $p = E/c$ is equivalent to Compton's result $p = h/\lambda$. For a further verification of the relationship between photon energy and momentum, see [link].

Note:
Photon Detectors
Almost all detection systems talked about thus far—eyes, photographic plates, photomultiplier tubes in microscopes, and CCD cameras—rely on particle-like properties of photons interacting with a sensitive area. A change is caused and either the change is cascaded or zillions of points are recorded to form an image we detect.

These detectors are used in biomedical imaging systems, and there is ongoing research into improving the efficiency of receiving photons, particularly by cooling detection systems and reducing thermal effects.

Example:
Photon Energy and Momentum
Show that $p=E/c$ for the photon considered in the [link].
Strategy
We will take the energy E found in [link], divide it by the speed of light, and see if the same momentum is obtained as before.
Solution
Given that the energy of the photon is 2.48 eV and converting this to joules, we get
Equation:

$$p=\frac{E}{c}=\frac{(2.48\text{ eV})(1.60\times10^{-19}\text{ J/eV})}{3.00\times10^{8}\text{ m/s}}=1.33\times10^{-27}\text{ kg}\cdot\text{m/s}.$$

Discussion
This value for momentum is the same as found before (note that unrounded values are used in all calculations to avoid even small rounding errors), an expected verification of the relationship $p=E/c$. This also means the relationship between energy, momentum, and mass given by $E^2=(\text{pc})^2+(\text{mc})^2$ applies to both matter and photons. Once again, note that p is not zero, even when m is.

Note:
Problem-Solving Suggestion
Note that the forms of the constants $h=4.14\times10^{-15}\text{ eV}\cdot\text{s}$ and $\text{hc}=1240\text{ eV}\cdot\text{nm}$ may be particularly useful for this section’s Problems and Exercises.

Section Summary

- Photons have momentum, given by $p=\frac{h}{\lambda}$, where λ is the photon wavelength.
- Photon energy and momentum are related by $p=\frac{E}{c}$, where $E=\text{hf}=\text{hc}/\lambda$ for a photon.

Conceptual Questions

Exercise:

Problem: Why are UV, x rays, and γ rays called ionizing radiation?

Exercise:

Problem:

How can treating food with ionizing radiation help keep it from spoiling? UV is not very penetrating. What else could be used?

Exercise:

Problem:

Some television tubes are CRTs. They use an approximately 30-kV accelerating potential to send electrons to the screen, where the electrons stimulate phosphors to emit the light that forms the pictures we watch. Would you expect x rays also to be created?

Exercise:

Problem:

Tanning salons use “safe” UV with a longer wavelength than some of the UV in sunlight. This “safe” UV has enough photon energy to trigger the tanning mechanism. Is it likely to be able to cause cell damage and induce cancer with prolonged exposure?

Exercise:

Problem:

Your pupils dilate when visible light intensity is reduced. Does wearing sunglasses that lack UV blockers increase or decrease the UV hazard to your eyes? Explain.

Exercise:

Problem:

One could feel heat transfer in the form of infrared radiation from a large nuclear bomb detonated in the atmosphere 75 km from you. However, none of the profusely emitted x rays or γ rays reaches you. Explain.

Exercise:

Problem: Can a single microwave photon cause cell damage? Explain.

Exercise:

Problem:

In an x-ray tube, the maximum photon energy is given by $\mathrm{hf} = \mathrm{qV}$. Would it be technically more correct to say $\mathrm{hf} = \mathrm{qV} + \mathrm{BE}$, where BE is the binding energy of electrons in the target anode? Why isn't the energy stated the latter way?

Exercise:

Problem:

Which formula may be used for the momentum of all particles, with or without mass?

Exercise:

Problem:

Is there any measurable difference between the momentum of a photon and the momentum of matter?

Exercise:

Problem:

Why don't we feel the momentum of sunlight when we are on the beach?

Problems & Exercises

Exercise:

Problem:

What is the energy in joules and eV of a photon in a radio wave from an AM station that has a 1530-kHz broadcast frequency?

Solution:

6.34×10^{-9} eV, 1.01×10^{-27} J

Exercise:

Problem:

(a) Find the energy in joules and eV of photons in radio waves from an FM station that has a 90.0-MHz broadcast frequency. (b) What does this imply about the number of photons per second that the radio station must broadcast?

Exercise:

Problem: Calculate the frequency in hertz of a 1.00-MeV γ-ray photon.

Solution:

2.42×10^{20} Hz

Exercise:

Problem:

(a) What is the wavelength of a 1.00-eV photon? (b) Find its frequency in hertz. (c) Identify the type of EM radiation.

Exercise:

Problem:

Do the unit conversions necessary to show that $\text{hc} = 1240 \text{ eV} \cdot \text{nm}$, as stated in the text.

Solution:
Equation:

$$\begin{aligned} \text{hc} &= \left(6.62607 \times 10^{-34}\text{ J} \cdot \text{s}\right)\left(2.99792 \times 10^{8}\text{ m/s}\right)\left(\frac{10^{9}\text{ nm}}{1\text{ m}}\right)\left(\frac{1.00000\text{ eV}}{1.60218 \times 10^{-19}\text{ J}}\right) \\ &= 1239.84\text{ eV} \cdot \text{nm} \\ &\approx 1240\text{ eV} \cdot \text{nm} \end{aligned}$$

Exercise:

Problem:

Confirm the statement in the text that the range of photon energies for visible light is 1.63 to 3.26 eV, given that the range of visible wavelengths is 380 to 760 nm.

Exercise:

Problem:

(a) Calculate the energy in eV of an IR photon of frequency 2.00×10^{13} Hz. (b) How many of these photons would need to be absorbed simultaneously by a tightly bound molecule to break it apart? (c) What is the energy in eV of a γ ray of frequency 3.00×10^{20} Hz? (d) How many tightly bound molecules could a single such γ ray break apart?

Solution:

(a) 0.0829 eV

(b) 121

(c) 1.24 MeV

(d) 1.24×10^{5}

Exercise:

Problem:

Prove that, to three-digit accuracy, $h = 4.14 \times 10^{-15}$ eV $\cdot$ s, as stated in the text.

Exercise:

Problem:

(a) What is the maximum energy in eV of photons produced in a CRT using a 25.0-kV accelerating potential, such as a color TV? (b) What is their frequency?

Solution:

(a) 25.0×10^{3} eV

(b) 6.04×10^{18} Hz

Exercise:

Problem:

What is the accelerating voltage of an x-ray tube that produces x rays with a shortest wavelength of 0.0103 nm?

Exercise:

Problem:

(a) What is the ratio of power outputs by two microwave ovens having frequencies of 950 and 2560 MHz, if they emit the same number of photons per second? (b) What is the ratio of photons per second if they have the same power output?

Solution:

(a) 2.69

(b) 0.371

Exercise:

Problem:

How many photons per second are emitted by the antenna of a microwave oven, if its power output is 1.00 kW at a frequency of 2560 MHz?

Exercise:

Problem:

Some satellites use nuclear power. (a) If such a satellite emits a 1.00-W flux of γ rays having an average energy of 0.500 MeV, how many are emitted per second? (b) These γ rays affect other satellites. How far away must another satellite be to only receive one γ ray per second per square meter?

Solution:

(a) $1.25 \times 10^{13}\ \mathrm{photons/s}$

(b) 997 km

Exercise:

Problem:

(a) If the power output of a 650-kHz radio station is 50.0 kW, how many photons per second are produced? (b) If the radio waves are broadcast uniformly in all directions, find the number of photons per second per square meter at a distance of 100 km. Assume no reflection from the ground or absorption by the air.

Exercise:

Problem:

How many x-ray photons per second are created by an x-ray tube that produces a flux of x rays having a power of 1.00 W? Assume the average energy per photon is 75.0 keV.

Solution:

8.33×10^{13} photons/s

Exercise:

Problem:

(a) How far away must you be from a 650-kHz radio station with power 50.0 kW for there to be only one photon per second per square meter? Assume no reflections or absorption, as if you were in deep outer space. (b) Discuss the implications for detecting intelligent life in other solar systems by detecting their radio broadcasts.

Exercise:

Problem:

Assuming that 10.0% of a 100-W light bulb's energy output is in the visible range (typical for incandescent bulbs) with an average wavelength of 580 nm, and that the photons spread out uniformly and are not absorbed by the atmosphere, how far away would you be if 500 photons per second enter the 3.00-mm diameter pupil of your eye? (This number easily stimulates the retina.)

Solution:

181 km

Exercise:

Problem: **Construct Your Own Problem**

Consider a laser pen. Construct a problem in which you calculate the number of photons per second emitted by the pen. Among the things to be considered are the laser pen's wavelength and power output. Your instructor may also wish for you to determine the minimum diffraction spreading in the beam and the number of photons per square centimeter the pen can project at some large distance. In this latter case, you will also need to consider the output size of the laser beam,

the distance to the object being illuminated, and any absorption or scattering along the way.

Exercise:

Problem:

(a) Find the momentum of a 4.00-cm-wavelength microwave photon. (b) Discuss why you expect the answer to (a) to be very small.

Solution:

(a) $1.66 \times 10^{-32}\ \mathrm{kg \cdot m/s}$

(b) The wavelength of microwave photons is large, so the momentum they carry is very small.

Exercise:

Problem:

(a) What is the momentum of a 0.0100-nm-wavelength photon that could detect details of an atom? (b) What is its energy in MeV?

Exercise:

Problem:

(a) What is the wavelength of a photon that has a momentum of $5.00 \times 10^{-29}\ \mathrm{kg \cdot m/s}$? (b) Find its energy in eV.

Solution:

(a) 13.3 μm

(b) 9.38×10^{-2} eV

Exercise:

Problem:

(a) A γ-ray photon has a momentum of $8.00 \times 10^{-21}\ \mathrm{kg \cdot m/s}$. What is its wavelength? (b) Calculate its energy in MeV.

Exercise:

Problem:

(a) Calculate the momentum of a photon having a wavelength of $2.50\ \mu\text{m}$. (b) Find the velocity of an electron having the same momentum. (c) What is the kinetic energy of the electron, and how does it compare with that of the photon?

Solution:

(a) $2.65 \times 10^{-28}\ \text{kg} \cdot \text{m/s}$

(b) 291 m/s

(c) electron $3.86 \times 10^{-26}\ \text{J}$, photon $7.96 \times 10^{-20}\ \text{J}$, ratio 2.06×10^{6}

Exercise:

Problem: Repeat the previous problem for a 10.0-nm-wavelength photon.

Exercise:

Problem:

(a) Calculate the wavelength of a photon that has the same momentum as a proton moving at 1.00% of the speed of light. (b) What is the energy of the photon in MeV? (c) What is the kinetic energy of the proton in MeV?

Solution:

(a) $1.32 \times 10^{-13}\ \text{m}$

(b) 9.39 MeV

(c) $4.70 \times 10^{-2}\ \text{MeV}$

Exercise:

Problem:

(a) Find the momentum of a 100-keV x-ray photon. (b) Find the equivalent velocity of a neutron with the same momentum. (c) What is the neutron’s kinetic energy in keV?

Exercise:

Problem:

Take the ratio of relativistic rest energy, $E = \gamma mc^2$, to relativistic momentum, $p = \gamma \text{mu}$, and show that in the limit that mass approaches zero, you find $E/p = c$.

Solution:

$E = \gamma mc^2$ and $P = \gamma mu$, so

Equation:

$$\frac{E}{P} = \frac{\gamma mc^2}{\gamma mu} = \frac{c^2}{u}.$$

As the mass of particle approaches zero, its velocity u will approach c, so that the ratio of energy to momentum in this limit is

Equation:

$$\lim_{m \to 0} \frac{E}{P} = \frac{c^2}{c} = c$$

which is consistent with the equation for photon energy.

Exercise:

Problem: Construct Your Own Problem

Consider a space sail such as mentioned in [link]. Construct a problem in which you calculate the light pressure on the sail in N/m^2 produced by reflecting sunlight. Also calculate the force that could be produced and how much effect that would have on a spacecraft. Among the things to be considered are the intensity of sunlight, its average wavelength, the number of photons per square meter this implies, the area of the space sail, and the mass of the system being accelerated.

Exercise:

Problem: Unreasonable Results

A car feels a small force due to the light it sends out from its headlights, equal to the momentum of the light divided by the time in which it is emitted. (a) Calculate the power of each headlight, if they exert a total force of 2.00×10^{-2} N backward on the car. (b) What is unreasonable about this result? (c) Which assumptions are unreasonable or inconsistent?

Solution:

(a) 3.00×10^{6} W

(b) Headlights are way too bright.

(c) Force is too large.

Glossary

photon momentum
: the amount of momentum a photon has, calculated by $p = \frac{h}{\lambda} = \frac{E}{c}$

Compton effect
: the phenomenon whereby x rays scattered from materials have decreased energy

The Connection Between Kinetic Energy and Momentum
Exercise:

UMASS AMHERST Instructor's Notes

Problem:

This section is also available as a video on the UMass Physics 13X YouTube page.The link can be found here.

This section is about a final point connecting the momentum and kinetic energies of particles with mass. This section assumes that you have already refreshed your memory on the definitions of momentum and energy from Physics 131 using the other resources provided.

We will now develop a useful relationship between momentum and kinetic energy. This is a useful relationship that we will use throughout this course.

By now you should have refreshed your memory and know that for a standard particle with mass, such as an electron, the momentum of the particle is

$$\overrightarrow{p} = m\overrightarrow{v}$$

and the kinetic energy of the particle is

$$K = \frac{1}{2}mv^2$$

. If you look at these two expressions, they are fairly similar, both involve the mass of the particle

$$m$$

and its velocity

$$v$$

.

Now there are some important differences. The momentum is a vector, including the direction of motion, whereas the kinetic energy is a scalar and is independent of the particles direction of motion. However, there's a useful way to relate these two.

Begin with the magnitude of the momentum, removing the vectors, in which case is just

$$p = mv$$

.

Square both sides of this expression so you have

$$p^2 = m^2 v^2$$

.

Now divide both sides of the expression by

$$2m$$

, so now you have

$$\frac{p^2}{2m} = \frac{1}{2} m v^2$$

.

And this is the kinetic energy.

The big punch line is that the kinetic energy of a particle with mass is

$$K = \frac{p^2}{2m}$$

. This is a useful expression that we'll be using throughout this course.

The Wave Nature of Matter

- Describe the Davisson-Germer experiment, and explain how it provides evidence for the wave nature of electrons.

De Broglie Wavelength

In 1923 a French physics graduate student named Prince Louis-Victor de Broglie (1892–1987) made a radical proposal based on the hope that nature is symmetric. If EM radiation has both particle and wave properties, then nature would be symmetric if matter also had both particle and wave properties. If what we once thought of as an unequivocal wave (EM radiation) is also a particle, then what we think of as an unequivocal particle (matter) may also be a wave. De Broglie's suggestion, made as part of his doctoral thesis, was so radical that it was greeted with some skepticism. A copy of his thesis was sent to Einstein, who said it was not only probably correct, but that it might be of fundamental importance. With the support of Einstein and a few other prominent physicists, de Broglie was awarded his doctorate.

De Broglie took both relativity and quantum mechanics into account to develop the proposal that *all particles have a wavelength*, given by **Equation:**

$$\lambda = \frac{h}{p} \text{ (matter and photons)},$$

where h is Planck's constant and p is momentum. This is defined to be the **de Broglie wavelength**. (Note that we already have this for photons, from the equation $p = h/\lambda$.) The hallmark of a wave is interference. If matter is a wave, then it must exhibit constructive and destructive interference. Why isn't this ordinarily observed? The answer is that in order to see significant interference effects, a wave must interact with an object about the same size as its wavelength. Since h is very small, λ is also small, especially for macroscopic objects. A 3-kg bowling ball moving at 10 m/s, for example, has

Equation:

$$\lambda = h/p = (6.63 \times 10^{-34}\ \text{J}\cdot\text{s})/[(3\ \text{kg})(10\ \text{m/s}\ )]= 2 \times 10^{-35}\ \text{m}.$$

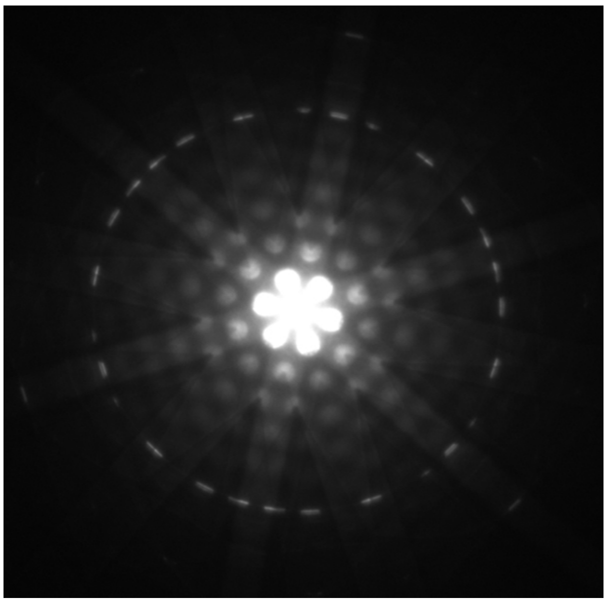
This means that to see its wave characteristics, the bowling ball would have to interact with something about 10^{-35} m in size—far smaller than anything known. When waves interact with objects much larger than their wavelength, they show negligible interference effects and move in straight lines (such as light rays in geometric optics). To get easily observed interference effects from particles of matter, the longest wavelength and hence smallest mass possible would be useful. Therefore, this effect was first observed with electrons.

American physicists Clinton J. Davisson and Lester H. Germer in 1925 and, independently, British physicist G. P. Thomson (son of J. J. Thomson, discoverer of the electron) in 1926 scattered electrons from crystals and found diffraction patterns. These patterns are exactly consistent with interference of electrons having the de Broglie wavelength and are somewhat analogous to light interacting with a diffraction grating. (See [link].)

Note:
Connections: Waves
All microscopic particles, whether massless, like photons, or having mass, like electrons, have wave properties. The relationship between momentum and wavelength is fundamental for all particles.

De Broglie's proposal of a wave nature for all particles initiated a remarkably productive era in which the foundations for quantum mechanics were laid. In 1926, the Austrian physicist Erwin Schrödinger (1887–1961) published four papers in which the wave nature of particles was treated explicitly with wave equations. At the same time, many others began important work. Among them was German physicist Werner Heisenberg

(1901–1976) who, among many other contributions to quantum mechanics, formulated a mathematical treatment of the wave nature of matter that used matrices rather than wave equations. We will deal with some specifics in later sections, but it is worth noting that de Broglie's work was a watershed for the development of quantum mechanics. De Broglie was awarded the Nobel Prize in 1929 for his vision, as were Davisson and G. P. Thomson in 1937 for their experimental verification of de Broglie's hypothesis.



This diffraction pattern was obtained for electrons diffracted by crystalline silicon. Bright regions are those of constructive interference, while dark regions are those of destructive interference. (credit: Ndthe, Wikimedia Commons)

Example:

Electron Wavelength versus Velocity and Energy
For an electron having a de Broglie wavelength of 0.167 nm (appropriate for interacting with crystal lattice structures that are about this size): (a) Calculate the electron’s velocity, assuming it is nonrelativistic. (b) Calculate the electron’s kinetic energy in eV.
Strategy
For part (a), since the de Broglie wavelength is given, the electron’s velocity can be obtained from $\lambda = h/p$ by using the nonrelativistic formula for momentum, $p = \mathrm{mv}$. For part (b), once v is obtained (and it has been verified that v is nonrelativistic), the classical kinetic energy is simply $(1/2)mv^2$.
Solution for (a)
Substituting the nonrelativistic formula for momentum ($p = \mathrm{mv}$) into the de Broglie wavelength gives
Equation:

$$\lambda = \frac{h}{p} = \frac{h}{\mathrm{mv}}.$$

Solving for v gives
Equation:

$$v = \frac{h}{m\lambda}.$$

Substituting known values yields
Equation:

$$v = \frac{6.63 \times 10^{-34}\ \mathrm{J \cdot s}}{(9.11 \times 10^{-31}\ \mathrm{kg})(0.167 \times 10^{-9}\ \mathrm{m})} = 4.36 \times 10^{6}\ \mathrm{m/s}.$$

Solution for (b)
While fast compared with a car, this electron’s speed is not highly relativistic, and so we can comfortably use the classical formula to find the electron’s kinetic energy and convert it to eV as requested.
Equation:

$$
\begin{aligned}
\text{KE} &= \frac{1}{2}mv^2 \\
&= \frac{1}{2}(9.11 \times 10^{-31}\ \text{kg})(4.36 \times 10^{6}\ \text{m/s})^2 \\
&= (86.4 \times 10^{-18}\ \text{J})\left(\frac{1\ \text{eV}}{1.602 \times 10^{-19}\ \text{J}}\right) \\
&= 54.0\ \text{eV}
\end{aligned}
$$

Discussion

This low energy means that these 0.167-nm electrons could be obtained by accelerating them through a 54.0-V electrostatic potential, an easy task. The results also confirm the assumption that the electrons are nonrelativistic, since their velocity is just over 1% of the speed of light and the kinetic energy is about 0.01% of the rest energy of an electron (0.511 MeV). If the electrons had turned out to be relativistic, we would have had to use more involved calculations employing relativistic formulas.

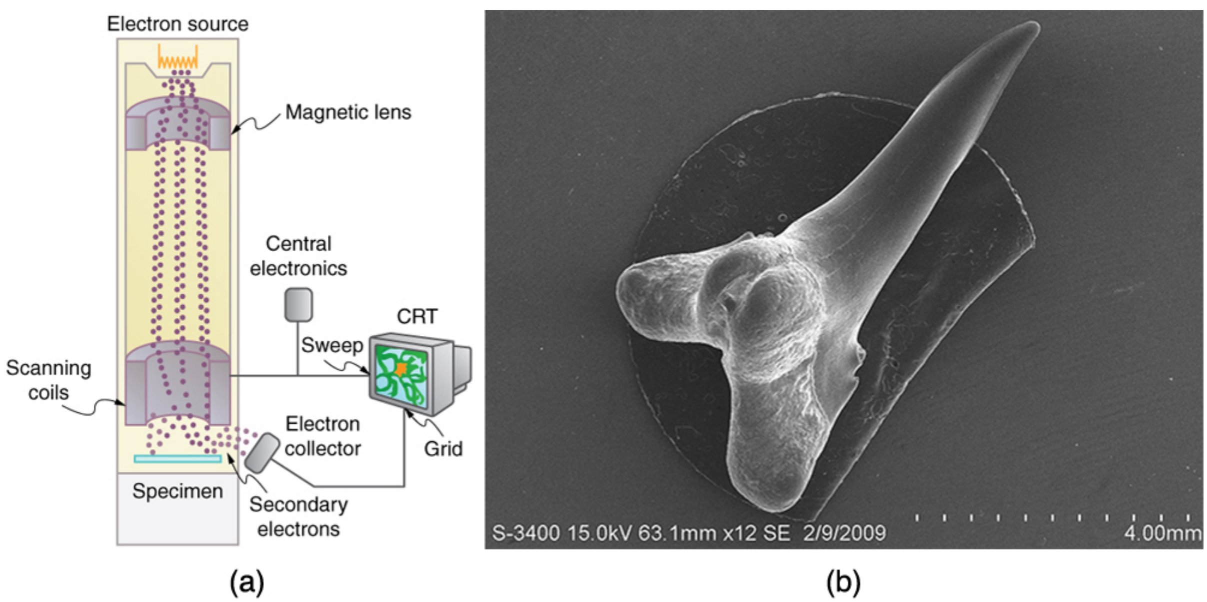
Electron Microscopes

One consequence or use of the wave nature of matter is found in the electron microscope. As we have discussed, there is a limit to the detail observed with any probe having a wavelength. Resolution, or observable detail, is limited to about one wavelength. Since a potential of only 54 V can produce electrons with sub-nanometer wavelengths, it is easy to get electrons with much smaller wavelengths than those of visible light (hundreds of nanometers). Electron microscopes can, thus, be constructed to detect much smaller details than optical microscopes. (See [link].)

There are basically two types of electron microscopes. The transmission electron microscope (TEM) accelerates electrons that are emitted from a hot filament (the cathode). The beam is broadened and then passes through the sample. A magnetic lens focuses the beam image onto a fluorescent screen, a photographic plate, or (most probably) a CCD (light sensitive camera), from which it is transferred to a computer. The TEM is similar to the optical microscope, but it requires a thin sample examined in a vacuum. However it can resolve details as small as 0.1 nm (10^{-10} m), providing magnifications

of 100 million times the size of the original object. The TEM has allowed us to see individual atoms and structure of cell nuclei.

The scanning electron microscope (SEM) provides images by using secondary electrons produced by the primary beam interacting with the surface of the sample (see [link]). The SEM also uses magnetic lenses to focus the beam onto the sample. However, it moves the beam around electrically to “scan” the sample in the *x* and *y* directions. A CCD detector is used to process the data for each electron position, producing images like the one at the beginning of this chapter. The SEM has the advantage of not requiring a thin sample and of providing a 3-D view. However, its resolution is about ten times less than a TEM.



Schematic of a scanning electron microscope (SEM) (a) used to observe small details, such as those seen in this image of a tooth of a *Himipristis*, a type of shark (b). (credit: Dallas Krentzel, Flickr)

Electrons were the first particles with mass to be directly confirmed to have the wavelength proposed by de Broglie. Subsequently, protons, helium nuclei, neutrons, and many others have been observed to exhibit

interference when they interact with objects having sizes similar to their de Broglie wavelength. The de Broglie wavelength for massless particles was well established in the 1920s for photons, and it has since been observed that all massless particles have a de Broglie wavelength $\lambda = h/p.$ The wave nature of all particles is a universal characteristic of nature. We shall see in following sections that implications of the de Broglie wavelength include the quantization of energy in atoms and molecules, and an alteration of our basic view of nature on the microscopic scale. The next section, for example, shows that there are limits to the precision with which we may make predictions, regardless of how hard we try. There are even limits to the precision with which we may measure an object's location or energy.

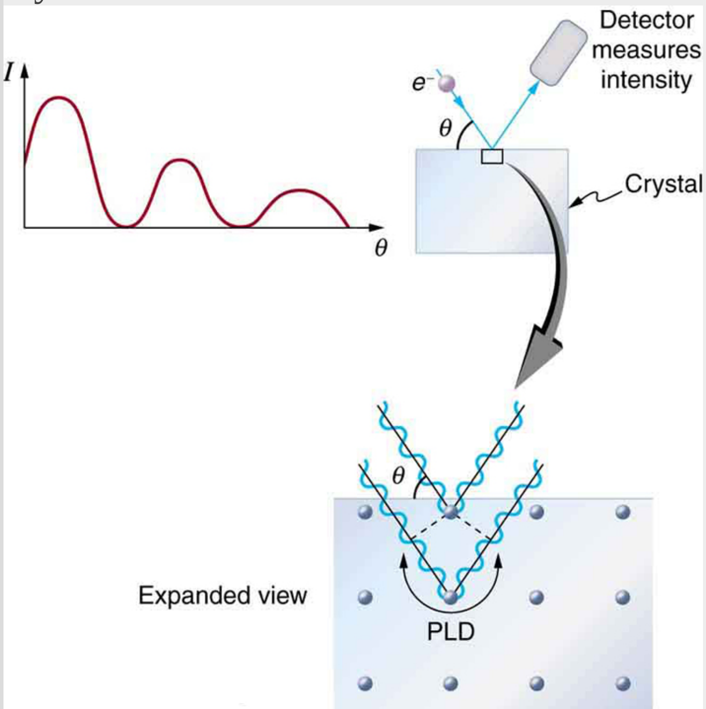
Note:

Making Connections: A Submicroscopic Diffraction Grating

The wave nature of matter allows it to exhibit all the characteristics of other, more familiar, waves. Diffraction gratings, for example, produce diffraction patterns for light that depend on grating spacing and the wavelength of the light. This effect, as with most wave phenomena, is most pronounced when the wave interacts with objects having a size similar to its wavelength. For gratings, this is the spacing between multiple slits.) When electrons interact with a system having a spacing similar to the electron wavelength, they show the same types of interference patterns as light does for diffraction gratings, as shown at top left in [link].

Atoms are spaced at regular intervals in a crystal as parallel planes, as shown in the bottom part of [link]. The spacings between these planes act like the openings in a diffraction grating. At certain incident angles, the paths of electrons scattering from successive planes differ by one wavelength and, thus, interfere constructively. At other angles, the path length differences are not an integral wavelength, and there is partial to total destructive interference. This type of scattering from a large crystal with well-defined lattice planes can produce dramatic interference patterns. It is called *Bragg reflection*, for the father-and-son team who first explored and analyzed it in some detail. The expanded view also shows the path-length differences and indicates how these depend on incident angle θ in a

manner similar to the diffraction patterns for x rays reflecting from a crystal.



The diffraction pattern at top left is produced by scattering electrons from a crystal and is graphed as a function of incident angle relative to the regular array of atoms in a crystal, as shown at bottom. Electrons scattering from the second layer of atoms travel farther than those scattered from the top layer. If the path length difference (PLD) is an integral wavelength, there is constructive interference.

Let us take the spacing between parallel planes of atoms in the crystal to be d. As mentioned, if the path length difference (PLD) for the electrons is a whole number of wavelengths, there will be constructive interference—that is, $\text{PLD} = n\lambda (n = 1, 2, 3, \ldots)$. Because $\text{AB} = \text{BC} = d \sin \theta$, we have constructive interference when $n\lambda = 2d \sin \theta$. This relationship is

called the *Bragg equation* and applies not only to electrons but also to x rays.
The wavelength of matter is a submicroscopic characteristic that explains a macroscopic phenomenon such as Bragg reflection. Similarly, the wavelength of light is a submicroscopic characteristic that explains the macroscopic phenomenon of diffraction patterns.

Section Summary

- Particles of matter also have a wavelength, called the de Broglie wavelength, given by $\lambda = \frac{h}{p}$, where p is momentum.
- Matter is found to have the same *interference characteristics* as any other wave.

Conceptual Questions

Exercise:

Problem:

How does the interference of water waves differ from the interference of electrons? How are they analogous?

Exercise:

Problem: Describe one type of evidence for the wave nature of matter.

Exercise:

Problem:

Describe one type of evidence for the particle nature of EM radiation.

Problems & Exercises

Exercise:

Problem:

At what velocity will an electron have a wavelength of 1.00 m?

Solution:

7.28×10^{-4} m

Exercise:

Problem:

What is the wavelength of an electron moving at 3.00% of the speed of light?

Exercise:

Problem:

At what velocity does a proton have a 6.00-fm wavelength (about the size of a nucleus)? Assume the proton is nonrelativistic. (1 femtometer = 10^{-15} m.)

Solution:

$6.62 \times 10^{7}\ \text{m/s}$

Exercise:

Problem:

What is the velocity of a 0.400-kg billiard ball if its wavelength is 7.50 cm (large enough for it to interfere with other billiard balls)?

Exercise:

Problem:

Find the wavelength of a proton moving at 1.00% of the speed of light.

Solution:

$1.32 \times 10^{-13}\ \text{m}$

Exercise:

Problem:

Experiments are performed with ultracold neutrons having velocities as small as 1.00 m/s. (a) What is the wavelength of such a neutron? (b) What is its kinetic energy in eV?

Exercise:

Problem:

(a) Find the velocity of a neutron that has a 6.00-fm wavelength (about the size of a nucleus). Assume the neutron is nonrelativistic. (b) What is the neutron's kinetic energy in MeV?

Solution:

(a) $6.62 \times 10^{7}\ \text{m/s}$

(b) $22.9\ \text{MeV}$

Exercise:

Problem:

What is the wavelength of an electron accelerated through a 30.0-kV potential, as in a TV tube?

Exercise:

Problem:

What is the kinetic energy of an electron in a TEM having a 0.0100-nm wavelength?

Solution:
Equation:15.1 keV

Exercise:

Problem:

(a) Calculate the velocity of an electron that has a wavelength of $1.00\ \mu\mathrm{m}$. (b) Through what voltage must the electron be accelerated to have this velocity?

Exercise:

Problem:

The velocity of a proton emerging from a Van de Graaff accelerator is 25.0% of the speed of light. (a) What is the proton’s wavelength? (b) What is its kinetic energy, assuming it is nonrelativistic? (c) What was the equivalent voltage through which it was accelerated?

Solution:

(a) 5.29 fm

(b) 4.70×10^{-12} J

(c) 29.4 MV

Exercise:

Problem:

The kinetic energy of an electron accelerated in an x-ray tube is 100 keV. Assuming it is nonrelativistic, what is its wavelength?

Exercise:

Problem: Unreasonable Results

(a) Assuming it is nonrelativistic, calculate the velocity of an electron with a 0.100-fm wavelength (small enough to detect details of a nucleus). (b) What is unreasonable about this result? (c) Which assumptions are unreasonable or inconsistent?

Solution:

(a) $7.28 \times 10^{12}\ \mathrm{m/s}$

(b) This is thousands of times the speed of light (an impossibility).

(c) The assumption that the electron is non-relativistic is unreasonable at this wavelength.

Glossary

de Broglie wavelength
: the wavelength possessed by a particle of matter, calculated by $\lambda = h/p$